

Vacuum Polarization around Black Holes and Quantum Hair¹

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Properties of black holes (BHs) with quantum hair [1] have been discussed recently [2]. The quantum hair is supported by virtual string loops surrounding the BH. In this talk, we examine the renormalized value of $\langle \phi^+ \phi \rangle$ and stress tensor for a scalar field in the Hartle-Hawking vacuum around Schwarzschild BHs in the presence of the virtual string loops.

First of all, consider an Abelian Higgs model in D-dimensional spacetime. Suppose that the Higgs scalar has the U(1) charge Ne , where N is an integer larger than one. Another complex, free, scalar field is introduced, which has the charge e . In this system, a vortex,

¹This talk is based on ref. [0].

which has (D-3)-dimensional extension, is made from the circular configuration of the Higgs field. Although this extended object (string) is invisible for a classical measurement, the additional complex scalar field feels the existence of the for the vortex quantum-mechanically.

A virtual loop of the string can be created around an O(D-1)-symmetric BH in the D-dimensional spacetime. The (D-2) dimensional world-sheet of the string can wrap the membrane of the event horizon of the BH. The virtual motion of the object induces a virtual electric field, though the field strength is confined at the wake of the loop if the string is assumed very thin.

At spatial infinity, a field strength takes a topologically-determined value in the presence of the virtual string world-sheet wrapping the horizon k -fold. We can simply express the field strength by the following gauge field:

$$A_\tau \rightarrow \frac{2\pi k}{N\hbar e\beta}, \quad (1)$$

where $\beta^{-1}=(D-3)\hbar c/(4\pi r_g)$ and r_g is the Schwarzschild radius.

The higher components in the Fourier expansion with respect to the periodic Euclidean time (τ) are irrelevant for the quantum nature of the

complex scalar field. We can also neglect other components of the gauge field, because their contribution to quantum effect is small at $r \gg r_g$.

Now we start with the estimation of the vacuum polarization of the complex scalar field with the charge e in the Schwarzschild spacetime with the back ground gauge field configuration (1).

The asymptotic value of $\langle \phi^+ \phi \rangle$ at spatial infinity in D-dimensional Schwarzschild spacetime with the back ground gauge configuration is [0]

$$\langle \phi^+ \phi \rangle_k \rightarrow \frac{\hbar \Gamma((D-2)/2)}{2\pi^{D/2} c (\beta \hbar c)^{D-2}} \sum_{n=1}^{\infty} \frac{\cos(2\pi kn/N)}{n^{D-2}}. \quad (2)$$

Next we examine the quantum energy-momentum tensor which comes from the polarization of the scalar field. The asymptotic value at spatial infinity in D-dimensional spacetime turns out to be

$$\langle T_{\nu}^{\mu} \rangle_k \rightarrow \frac{2\hbar \Gamma(D/2)}{\pi^{D/2} c (\beta \hbar c)^D} \sum_{n=1}^{\infty} \frac{\cos(2\pi kn/N)}{n^D} \text{diag.}(- (D-1), 1, \dots, 1). \quad (3)$$

Finally, we consider the sum over the possible configurations. If the system including the BH has a charge Q , then the expectation value for $\langle \phi^+ \phi \rangle$ can be presented as

$$\langle \phi^+ \phi \rangle_Q \rightarrow \frac{1}{Z_Q} \sum_{k=-\infty}^{\infty} \exp\left(-i \frac{2\pi k Q}{N\hbar e}\right) \langle \phi^+ \phi \rangle_k e^{-S_{st.}/\hbar}, \quad (4)$$

where $S_{st.}$ is the action for the extended object (string) and can be approximated using the tension κ as

$$S_{st.} \approx \kappa |k| \frac{2\pi^{(D-1)/2}}{\Gamma((D-1)/2)} r_g^{D-2} \equiv \kappa |k| A. \quad (5)$$

Z_Q is defined as to keep the identity $\langle 1 \rangle_Q = 1$. $\langle T^\mu_\nu \rangle_Q$ can be defined in the same manner.

Then the asymptotic value of $\langle \phi^+ \phi \rangle$ at spatial infinity for the D -dimensional case is approximated by

$$\langle \phi^+ \phi \rangle_Q \rightarrow \frac{\hbar \Gamma((D-2)/2) \zeta(D-2)}{2\pi^{D/2} c(\beta \hbar c)^{D-2}} \times f_{D-2}(Q/\hbar e, N, \kappa A), \quad (6)$$

and similarly $\langle T^\mu_\nu \rangle_Q$ at infinity is given by

$$\langle T^\mu_\nu \rangle_Q \rightarrow \frac{2\hbar \Gamma(D/2)}{\pi^{D/2} c(\beta \hbar c)^D} f_D(Q/\hbar e, N, \kappa A) \cdot \text{diag.}(- (D-1), 1, \dots, 1), \quad (7)$$

where

$$f_x = \frac{1}{2\zeta(x)} \sum_{n=1}^{\infty} \frac{1}{n^x} \left\{ \frac{\cosh \kappa A - \cos(2\pi Q/N\hbar e)}{\cosh \kappa A - \cos(2\pi/N(Q/\hbar e - n))} + \frac{\cosh \kappa A - \cos(2\pi Q/N\hbar e)}{\cosh \kappa A - \cos(2\pi/N(Q/\hbar e + n))} \right\}. \quad (8)$$

For large κA , we obtain

$$f_x \approx 1 - \frac{2}{\zeta(x)} e^{-\kappa A} \cos \frac{2\pi Q}{N\hbar e} \sum_{n=1}^{\infty} \frac{1}{n^x} \frac{\sin^2 n\pi}{N}. \quad (9)$$

We have found that the amount of the Hawking radiation of the bose gas is suppressed if the total charge of the system vanishes ($Q=0$). (but it never becomes negative value.) Provided that the extrapolation to the region of small κA is permitted, the rise in the Hawking temperature may take place in the presence of quantum hair.

There seems to be much difficulty in the thermodynamical interpretation of the system, because the proper temperature of BH (β^{-1}) and the temperature of the charged boson gas take different values; the equilibrium of the total system is not attained in general, according to our result.

In this talk, we have studied the quantum property of the BHs with

virtual loops of strings. Numerical calculations for the precise value of physical quantities near a BH with virtual string loops will be necessary for the future analyses.

References

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