Multi-graviton theory, a latticized dimension, and the cosmological constant [1]

Nahomi Kan[2], Kiyoshi Shiraishi[3]

Graduate School of Science and Engineering, Yamaguchi University, Yoshida, Yamaguchi-shi, Yamaguchi 753-8512, Japan

Abstract

Beginning with the Pauli-Fierz theory, we construct a model for multi-graviton theory. Couplings between gravitons belonging to nearest-neighbor "theory spaces" lead to a discrete mass spectrum. Our model coincides with the Kaluza-Klein theory whose fifth dimension is latticized. We evaluate one-loop vacuum energy in models with a circular latticized extra dimensions as well as with compact continuous dimensions. We find that the vacuum energy can take a positive value, if the dimension of the continuous space time is $6, 10, \ldots$. Moreover, since the amount of the vacuum energy can be an arbitrary small value according to the choice of parameters in the model, our models is useful to explain the small positive dark energy in the present universe.

1 INTRODUCTION AND SUMMARY

It is well known that the weak-field limit of the Einstein gravity reduces to the theory of a single massless spin-two field [1]. We call the second-order symmetric tensor field $h_{\mu\nu}$, which represents deviation from a flat space ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$), as a "graviton". Multi-graviton theories have been studied be Boulanger *et al.* [2, 3]. They have shown that there is no consistent interaction of gravitons and the possible action is a sum of replicated Pauli-Fierz actions [4].

We consider a model for a multi-graviton theory with nearest-neighbor coupling in the theory space. To be precise, the gravitons are not interacting each other in our model, but they have a discrete mass spectrum. The simplest model is explained in Sec. 2. In the limit of a large number of gravitons, the mass spectrum of gravitons resembles that of the Kaluza-Klein (KK) theory. Therefore our model is equivalent to the KK theory with a discretized dimension.

The emergence mechanisms of a space dimension have been suggested in refs. [5, 6, 7], which is in the stream of dimensional deconstruction [8, 9]. Recently the authors have been informed of ref. [10], in which the gravity with the theory space is also argued. Although our model for the gravitation theory with a discretized dimension is very simple, it may be a toy model adequate for studying qualitative features of more complicated theory and mechanisms included in it.

For example, we evaluate the vaccum energy in our model in Sec. 3. We also consider a model in the continuous $(4+\delta)$ dimensional space-time with a discretized dimension. We show that an arbitrary small amount of positive vacuum energy can be obtained if $(4+\delta) = 6, 10, \ldots$.

2 MULTI-GRAVITON THEORY AND A LATTICIZED DIMENSION

We consider a multi-graviton (N-graviton) theory [2, 3] given by the lagrangian

$$\bar{\mathcal{L}}_0 = \sum_{k=1}^{N} \left[-\frac{1}{2}\partial_\lambda h^k_{\mu\nu}\partial^\lambda h^{k\,\mu\nu} + \partial_\lambda h^{k\,\lambda}{}_\mu \partial_\nu h^{k\,\nu\mu} - \partial_\mu h^{k\,\mu\nu}\partial_\nu h^k + \frac{1}{2}\partial_\lambda h^k \partial^\lambda h^k \right] , \tag{1}$$

[1] This work is based on gr-qc/0212113.

[2] E-mail:b1834@sty.cc.yamaguchi-u.ac.jp

[3] E-mail:shiraish@sci.yamaguchi-u.ac.jp

where $h \equiv h^{\mu}_{\mu}$. It can be said that a massless graviton lives in each "theory space", which is labeled by k.

Instead of adding the mere mass term to the lagrangian (1), we put the term including couplings of other gravitons with the nearest-neighbor suffix:

$$\begin{aligned}\bar{\mathcal{L}}_m &= \bar{\mathcal{L}}_0 - \frac{m^2}{2}\sum_{k=1}^{N}\left[(h^k_{\mu\nu} - h^{k+1}_{\mu\nu})^2 - (h^k - h^{k+1})^2\right] \\ &\quad -2\sum_{k=1}^{N}\left[m(A^{k\,\mu} - A^{k-1\,\mu}) + \partial^{\mu}\varphi^k)\right](\partial^{\nu}h^k_{\mu\nu} - \partial_{\mu}h^k) - \frac{1}{2}\sum_{k=1}^{N}(\partial_{\mu}A^k_{\nu} - \partial_{\nu}A^k_{\mu})^2 \,, \end{aligned} \tag{2}$$

where a constant m corresponds to the mass of graviton. We have omitted here the gauge fixing term as well as the ghost term. In this notation, one should read as $h^{k+N} = h^k$, $h^0 = h^N$, $h^{N+1} = h^1$, and so on. The multi-graviton lagrangian (2) is invariant under the following transformations:

$$\begin{aligned} h^k_{\mu\nu} &\to h^k_{\mu\nu} + \partial_{\mu}\xi^k_{\nu} + \partial_{\nu}\xi^k_{\mu} \,, \\ A^k_{\mu} &\to A^k_{\mu} + m\xi^k_{\mu} - m\xi^{k+1}_{\mu} - \partial_{\mu}\zeta^k \,, \\ \varphi^k &\to \varphi^k + m\zeta^k - m\zeta^{k-1} \,. \end{aligned} \tag{3}$$

As in the case with 5D QED studied by Hill and Leibovich [11], $N-1$ massive gravitons and one massless graviton are described by the lagrangian. The eigenvalues for the mass matrix are

$$M_p^2 = 4m^2\sin^2\left(\frac{\pi p}{N}\right) \qquad (p = 1, \ldots, N) \,. \tag{4}$$

Massive vector and scalar field are absorbed by the massive graviton fields by means of the transformations (3) and then they have correct spin-degree of freedom, five.

3 VACUUM ENERGY

In this section, we evaluate the one-loop vacuum energy density in the model of the previous section. We will also offer an extension of the model, which includes both latticized and continuous extra dimensions, and the vacuum energy is estimated in the model.

Recently, the scenarios explaining the small positive cosmological constant in the present universe (10^{-47} GeV^4, according to [12]) are discussed through the study of the Casimir-like energy in the higher-dimensional theories [13, 14]. Because the latticized model would have different UV behavior from that of continuum model, we expect a novel expression of the cosmological constant.

First, we consider one scalar degree of freedom which has the mass spectrum discussed in the previous section. Then the vacuum energy density in the one-loop calculation is written by the zeta-function technique [15] as

$$\frac{1}{2}\ln\det[-\nabla^2 + M_p^2] = -\frac{1}{2}\frac{d\zeta(s)}{ds}\bigg|_{s=0} \,, \tag{5}$$

where

$$\zeta(s) = \frac{\mu^{2s}}{16\pi^2\Gamma(s)}\sum_p\int_0^{\infty} dt\, t^{s-3}\exp(-M_p^2 t) \,, \tag{6}$$

with

$$M_p^2 = 4m^2\sin^2\left(\frac{\pi p}{N}\right) \,. \tag{7}$$

Here the constant μ has the dimension of mass.

The calculation of the zeta function can be done by the similar method used in refs. [16, 17]. Therefore the zeta function is written as

$$\zeta(s) = \frac{Nm^4}{\pi^2}\left(\frac{\mu^2}{4m^2}\right)^s\frac{\Gamma(\frac{5}{2}-s)}{\sqrt{\pi}}\left[\frac{1}{(s-1)(s-2)\Gamma(3-s)} + \frac{2}{\Gamma(s)}\sum_{q=1}^{\infty}\frac{\Gamma(qN-2+s)}{\Gamma(qN+3-s)}\right] \,. \tag{8}$$

Now we obtain

$$-\left.\frac{d\zeta(s)}{ds}\right|_{s=0} = -\frac{3Nm^4}{16\pi^2}\ln\left(\frac{\bar{\mu}^2}{4m^2}\right) - \frac{3m^4}{2\pi^2}\sum_{q=1}^{\infty}\frac{1}{q(q^2N^2-1)(q^2N^2-4)}\,, \tag{9}$$

where

$$\ln\left(\frac{\bar{\mu}^2}{4m^2}\right) = \ln\left(\frac{\mu^2}{4m^2}\right) + \psi(3) - \psi(5/2) + \frac{3}{2}\,. \tag{10}$$

Because there are five states in each mass level, the vacuum energy in the multi-graviton model in the previous section is

$$V = V_R - \frac{15Nm^4}{32\pi^2}\ln\left(\frac{\bar{\mu}^2}{4m^2}\right) - \frac{15m^4}{4\pi^2}\sum_{q=1}^{\infty}\frac{1}{q(q^2N^2-1)(q^2N^2-4)}\,, \tag{11}$$

where $V_R(\bar{\mu})$ is the renormalized vacuum energy.

According to ref. [18], We require the invariance of V with respect to the scale $\bar{\mu}$ and we choose $V_R(\bar{\mu}_R) = 0$ for a normalization point $\bar{\mu}_R$. Then we find

$$V = -\frac{15Nm^4}{32\pi^2}\ln\left(\frac{\bar{\mu}_R^2}{4m^2}\right) - \frac{15m^4}{4\pi^2}\sum_{q=1}^{\infty}\frac{1}{q(q^2N^2-1)(q^2N^2-4)}\,. \tag{12}$$

Unfortunately, this vacuum energy density is negative, for $\bar{\mu}_R$ is considered as a cutoff due to a scale of a new physics, $\bar{\mu}_R > m$. The boundary condition $V_R(\bar{\mu}_R) = 0$ can be interpreted as the expectation that the new physics forces the vacuum energy or cosmological constant to vanish at such a high-energy scale.

Let us consider the extension of our model to the one with $4+\delta$ dimensional continuous spacetime. Here the extra δ dimensional space is taken as $(S^1)^\delta$ with a common radius b. One can regard the new model as the theory with a latticized circle and δ continuous circle. In this model the mass spectrum of the gravitons is expressed as

$$M^2_{p,\{\mathbf{n}\}} = 4m^2\sin^2\left(\frac{\pi p}{N}\right) + \frac{\mathbf{n}^2}{b^2}\,, \tag{13}$$

where $\mathbf{n} = (n_1, n_2, \ldots, n_\delta)$ and the components take integer values.

Using the Poisson resummation formula, we can express the zeta function as

$$\begin{aligned}\zeta(s) \;=\; & \frac{\mu^{2s}N(2\pi b)^\delta}{(4\pi)^{2+\delta/2}\Gamma(s)}\sum_{\mathbf{n}}\left[\int_0^\infty dt\, t^{s-\delta/2-3}\exp\left(-2m^2t - \pi^2b^2\mathbf{n}^2/t\right)I_0(2m^2t)\right.\\ & \left. +2\sum_{q=1}^{\infty}\int_0^\infty dt\, t^{s-\delta/2-3}\exp\left(-2m^2t - \pi^2b^2\mathbf{n}^2/t\right)I_{qN}(2m^2t)\right]\,.\end{aligned} \tag{14}$$

If δ is odd, all the term in the parentheses are finite and lead to no logarithm term in the vacuum energy. The vacuum energy can be found to be finite and negative in this case, as in the case of the KK theory with torus compactification. Thus we assume that δ is even.

For even δ, in the limit $s \to 0$, only the first term with $\mathbf{n} = \mathbf{0}$ is divergent in the parentheses on the right hand side of (14) if N is larger than $2+\delta/2$. Thus, after taking account of the degrees of freedom, the vacuum energy density in our extended model takes the form

$$V = V_R - (-1)^{\delta/2}\frac{(5+\delta)(2+\delta)Nm^4}{4\pi^2}(4\pi m^2b^2)^{\delta/2}\frac{\Gamma\left(\frac{5+\delta}{2}\right)}{\sqrt{\pi}\Gamma\left(3+\frac{\delta}{2}\right)^2}\ln\left(\frac{\bar{\mu}^2}{4m^2}\right) - \cdots\,, \tag{15}$$

where

$$\ln\left(\frac{\bar{\mu}^2}{4m^2}\right) = \ln\left(\frac{\mu^2}{4m^2}\right) + 2\psi(3+\delta/2) - \psi(5/2+\delta/2) + \gamma\,, \tag{16}$$

(γ is the Euler's constant) and ... means the positive finite terms.

As seen from the sign of logarithmic term, we can obtain the positive vacuum energy after taking account of the renormalization discussed above; in other words, we have the positive cosmological constant when $\delta = 4k + 2$ ($k = 0, 1, 2, \ldots$), such as $\delta = 2$ ($D = 4 + \delta = 6$), $\delta = 6$ ($D = 10$) , and so on.

We will further investigate the possibility of the arbitrarily small cosmological constant. Since the contribution from the finite q terms in the zeta function can be estimated as at most $O(N^{-4})$, it can be neglected if a large N is assumed. Futher if $mb \ll 1$, we find

$$\begin{aligned} V &\approx \frac{(5+\delta)(2+\delta)Nm^4}{4\pi^2}(4\pi m^2 b^2)^{\delta/2}\frac{\Gamma\left(\frac{5+\delta}{2}\right)}{\sqrt{\pi}\Gamma\left(3+\frac{\delta}{2}\right)}\ln\left(\frac{\bar{\mu}_R^2}{4m^2}\right) \\ &- \frac{(5+\delta)(2+\delta)N}{4\pi^{(4+\delta)/2}(2\pi b)^4}\Gamma\left(\frac{4+\delta}{2}\right)\sum_{\mathbf{n}\neq 0}\frac{1}{(\mathbf{n}^2)^{(4+\delta)/2}} \qquad (mb \ll 1)\,. \end{aligned} \tag{17}$$

Therefore when $(2\pi mb)^{4+\delta}\ln(\bar{\mu}_R/m) \approx O(1)$, an arbitrary small vacuum energy can be obtained. Note that the condition does not include N in this approximation.

The difficulty of the present model in this section is the problem of tunning the scale of the extra dimensions. The model does not provide a mechanism to determine the value of b, because there is no minimum value of V as a function of b. We should consider either the value of b is due to the other mechanism or b takes an appropriate value only in the present universe. For the latter case, the motion of b may play a role of quintessence or dark energy, as similar to the suggestion in ref. [19]. This possibility will be considered elsewhere.

We only considered the vacuum energy at the one-loop level. Even at the same level, the correction to the Newton constant may also be induced in general. This would be related with the restriction or determination of the scale of the extra dimensions as well as the scale of m.

We should investigate the nature of coupling to the matter fields and back reaction problem in the case with a small cosmological constant. Through such studies, the case with strong gravity will also be revealed.

Finally, we remember that we have not treated the hierarchy problem in this paper. It will be clarified by the further study whether incorporating supersymmetry (and its break-down) into the model is necessary or not.

References

[1] B. Hatfield (ed.), *"Feynman Lectures on Gravitation"* (1995, Addison-Wesley, Reading, Mass.).

[2] N. Boulanger, hep-th/0111216.

[3] N. Boulanger, T. Damour, L. Gualtieri and M. Henneaux, *Nucl. Phys.* **B597** (2001) 127.

[4] M. Fierz and W. Pauli, *Proc. Roy. Soc. Lond.* **A173** (1939) 211.

[5] A. Sugamoto, *Prog. Theor. Phys.* **107** (2002) 793; hep-ph/0210235.

[6] M. Bander, *Phys. Rev.* **D64** (2001) 105021.

[7] V. Jejjala, R. G. Leigh and D. Minic, hep-th/0212057.

[8] N. Arkani-Hamed, A. G. Cohen and H. Georgi, *Phys. Rev. Lett.* **86** (2001) 4757; *Phys. Lett.* **B513** (2001) 232.

[9] C. T. Hill, S. Pokorski and J. Wang, *Phys. Rev.* **D64** (2001) 105005.

[10] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, hep-th/0210184.

[11] C. T. Hill and A. K. Leibovich, *Phys. Rev.* **D66** (2002) 016006, hep-ph/0205057.

[12] V. Sahni, *Class. Quant. Grav.* **19** (2002) 3435.

[13] A. Gupta, hep-th/0210069.

[14] K. A. Milton, hep-ph/0210170 and hep-th/0107241; K. A. Milton, R. Kantowski, C. Kao and Y. Wang, *Mod. Phys. Lett.* **A16** (2001) 2281, hep-ph/0105250.

[15] S. W. Hawking, *Commun. Math. Phys.* **55** (1977) 133.

[16] K. Shiraishi, K. Sakamoto and N. Kan, *"Shape of Deconstruction"*, hep-ph/0209126, *J. Phys.* **G** to appear.

[17] N. Kan, K. Sakamoto and K. Shiraishi, *"Deconstructing Scalar QED at Zero and Finite Temperature"*, hep-th/0209096.

[18] I. O. Cherednikov, *Acta Phys. Pol.* **33** (2002) 1973, astro-ph/011287; gr-qc/0205017.

[19] I. K. Wehus and F. Ranvdal, hep-ph/0210292.