

Scattering of Maximally-Charged Dilaton Black Holes^{1,2}

Kiyoshi Shiraishi³

Akita Junior College
Shimokitade-Sakura, Akita-shi, Akita 010, JAPAN

The action for the Einstein-Maxwell-Dilaton system we consider in (N+1) dimensions is: [1]

$$S = \int d^{N+1}x \frac{\sqrt{-g}}{16\pi} \left[R - \frac{4}{N-1} (\nabla\phi)^2 - e^{-(4a/(N-1))\phi} F^2 \right] \quad (N \geq 3), \quad (1)$$

where we fix the Newton constant to unity. The coupling constant a is the parameter which determines the strength of the coupling between the Maxwell field F and the dilaton field ϕ .

It is known that static forces among extreme black holes in this system are totally cancelled with one another. Therefore, there can exist multi-centered static solutions describing many-body systems of extreme charged black holes. We find n -black-hole solution in the following form:

$$ds^2 = - U^2(\vec{x}) dt^2 + U^{2/(N-2)}(\vec{x}) d\vec{x}^2, \quad (2)$$

where

$$U(\vec{x}) = \left\{ F(\vec{x}) \right\}^{(N-2)/(N-2+a^2)}, \quad (3)$$

¹Talk given at the Workshop on General Relativity and Gravitation (Waseda Univ., Tokyo, Japan), 18-20 January 1993.

²This talk is based on refs. [0].

³e-mail: g00345@jpnac.bitnet. or g00345@sinet.ad.jp.

and

$$F(\vec{x}) = 1 + \sum_{i=1}^n \frac{\mu_i}{(N-2)|\vec{x} - \vec{x}_i|^{N-2}} . \quad (4)$$

Using these expressions, the (Maxwell) vector one-form and the dilaton configuration are written as

$$A = \pm \sqrt{\frac{N-1}{2(N-2+a^2)}} [1 - \{F(\vec{x})\}^{-1}] dt , \quad (5)$$

and

$$e^{-\{4a/(N-1)\}\phi} = \{F(\vec{x})\}^{2a^2/(N-2+a^2)} . \quad (6)$$

In this solution, the asymptotic value of ϕ is fixed to be zero.

If we set $a=0$ in the solution, we find that the solution reduces to the Papapetrou-Majumdar-Myers solution in $(1+N)$ dimensions [2].

We anticipate that an addition of a small amount of kinetic energy to this static system can be treated by perturbation. This also presents the situation that radiation reactions can be ignored. We apply the method of Ferrell and Eardley [3] to the system of extreme charged dilaton black holes to get the information on the slow motion of the holes at arbitrary distances.

We first calculate the classical fields in the presence of the slow motion of the black-hole sources perturbatively. Secondly substituting the perturbative solutions into the action for the classical field, we obtain the effective action for maximally charged dilaton black holes as point sources.

Synthesizing all the contributions of small velocity perturbation, we obtain the effective lagrangian up to $O(v^2)$ for n maximally charged dilaton black holes:

$$L = - \sum_{i=1}^n m_i + \sum_{i=1}^n \frac{1}{2} m_i (\vec{v}_i)^2 + \frac{(N-1)(N-a^2)}{16\pi(N-2+a^2)^2} \int d^N x \{F(\vec{x})\}^{2(1-a^2)/(N-2+a^2)} \sum_{i,j} \frac{(\vec{n}_i \cdot \vec{n}_j) \vec{v}_i \cdot \vec{v}_j \mu_i \mu_j}{2|\vec{r}_i|^{N-1} |\vec{r}_j|^{N-1}} , \quad (7)$$

where $\vec{r}_i = \vec{x} - \vec{x}_i$ and $\vec{n}_i = \vec{r}_i / |\vec{r}_i|$. $F(x)$ is defined by (3).

The integration must be carried out with care for the regularization

of divergences and seemingly-divergent integrals.

If we set the dilaton coupling to zero and set $N=3$ in the above expression, we reproduce the result of ref. [3]. Furthermore, in the large-separation limit (where we approximate $F(x)$ by 1), we find exactly the same result obtained by the Lienard-Wiechert method [0].

An interesting point we soon become aware of is the existence of many-body interactions. In general, we obtain infinite species of many-body interactions by expanding the function $F(x)$. Some special cases arise: (1) when $a^2=0$ and $N=3$, the black holes are governed only by two-body, three-body, and four-body interactions [3], (2) when $a^2=0$ and $N=4$, there are only two-body and three-body interactions, (3) when $a^2=1$ (and any N), there are only two-body interactions, and (4) when $a^2=N$, there is no interactions to this order.

The slow motion of classical lumps or solitons in many kind of field theoretical models is expressed by geodesic motion on the moduli space, which is the space of the parameters in the static configuration.

In the presence case, the moduli space metric can be obtained from the expression (7). Now we further focus our attention on the two-body problem. Then the metric of the 2-dimensional moduli space for the two-body system turns out to be

$$ds_{MS}^2 = \gamma(r) \{ dr^2 + r^2 d\Omega_{N-1}^2 \} , \quad (8)$$

where

$$\begin{aligned} \gamma(r) = & \frac{A_{N-1}(N-1)}{8\pi(N-2+a^2)} \times \left[\frac{\mu_a \mu_b}{\mu_a + \mu_b} + \mu_a \left(1 + \frac{\mu_b}{(N-2)r^{N-2}} \right)^{(N-a^2)\gamma(N-2+a^2)} \right. \\ & \left. + \mu_b \left(1 + \frac{\mu_a}{(N-2)r^{N-2}} \right)^{\left(\frac{N-a^2}{(N-2+a^2)} \right) \mu_a - \mu_b} - \frac{N-a^2}{N-2+a^2} \frac{\mu_a \mu_b}{(N-2)r^{N-2}} \right] \end{aligned} \quad (9)$$

Moreover, we consider two-dimensional intersection of the moduli space for simplicity. The two parameters are the distance between two black holes (r) and the azimuthal angle (θ) on the scattering plane.

The shape of this surface of the moduli space depends on the spatial dimension N as well as a^2 . The reduced metric can be written in the form:

$$\begin{aligned}
ds_{MS}^2 &= \gamma(r) \{ dr^2 + r^2 d\theta^2 \} \\
&= h(R) dR^2 + R^2 d\theta^2 \\
&= dR^2 + dz^2(R) + R^2 d\theta^2
\end{aligned}
\tag{10}$$

where $R(r)$ is a new radial coordinate. We can realize the surface immersed in the three dimensional Euclidean space spanned by the coordinates R , θ , and $z(R)$. The surfaces for some case are sketched in Figs. 1 and 2. The "throat" of the surface is located at the distance where $h(R(r))$ diverges.

Suppose that the two black holes approach each other from spatial infinity with low relative velocity and impact parameter b , the two black holes coalesce if b is smaller than a certain critical value which is in the same order of the radius of the throat. If $a^2 > (N-2)^2/N$, moduli surface has no throat, and then two black holes never coalesce.

For $N=3$ and $a^2=1$, The black holes never coalesce according to the geodesic approximation on the moduli space. The differential cross section in this $N=3$ and $a^2=1$ case turns out to be written in the form of the Rutherford scattering:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \frac{M^2}{\sin^4(\Theta/2)} \tag{11}$$

This behavior can be expected by observing the geometry of the moduli space -- deficit angle = π near the origin for $N=3$ and $a^2=1$.

To summarize: We have studied the interaction among the maximally charged dilaton black holes in the low-velocity limit.

The nature of the interaction depends on the value of the dilaton coupling "a", in quality as well as in quantity.

$a=1$ is a special value, in any dimensions, for realizing a simple 2-body interactions.

There is another critical value, for coalescence of two maximally charged dilaton black holes: it is $a^2=(N-2)^2/N$. (for $N=3$, $a^2=1/3$). The global structure of moduli space depends on a^2 .

The geodesic approximation has been justified in the small mass and low velocity limit, by using the test particle/wave analyses. The

quantization on the moduli space have also been examined. For further detail, see refs. [0]. Other aspects on charged dilaton black holes are studied in refs. [00].

References

- [0] "Multi-Centered Solution for Maximally CDBHs in Arbitrary Dimensions", to appear in J. Math. Phys. 34; "Moduli Space Metric for Maximally CDBHs", AJC-HEP-8, to appear in Nucl. Phys. B; "Classical and Quantum Scattering of Maximally CDBHs", AJC-HEP-10, to appear in Int. J. Mod. Phys. D.
- [00] "Multi-Black Hole Solutions in Cosmological Einstein-Maxwell-Dilaton Theory", AJC-HEP-15; "Spinning a CDBH", Phys. Lett. A 166 (92) 298; "Superradiance from a CDBH", Mod. Phys. Lett. A7 (92) 3449; "Quantum Effects near CDBHs", Mod. Phys. Lett. A7 (92) 3569; "Motion of Test Particles around a CDBH", AJC-HEP-13.
- [1] G. W. Gibbons and K. Maeda, Nucl. Phys. B298 (1988) 741.
D. Garfinkle, G. Horowitz and A. Strominger, Phys. Rev. D43 (1991) 3140.
- [2] A. Papapetrou, Proc. R. Irish Acad. A51 (1947) 191.
S. D. Majumdar, Phys. Rev. 72 (1947) 930.
R. C. Myers, Phys. Rev. D35 (1987) 455.
- [3] R. C. Ferrell and D. M. Eardley, Phys. Rev. Lett. 59 (1987) 1617.

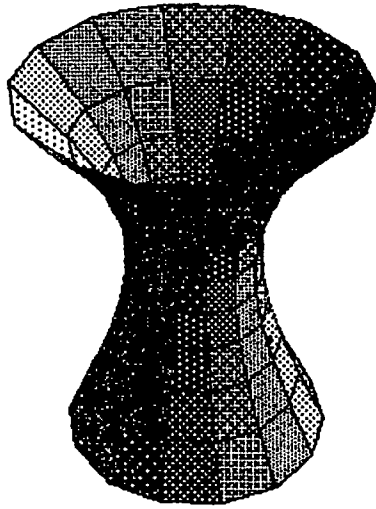
Figure Captions

Fig. 1: The surface of the moduli space for the two-body system of extreme charged dilaton black holes in three dimensional space ($N=3$). (a) for $a=0$, (b) for $a^2=1/3$, and (c) for $a^2=1$.

Fig. 2: The surface of the moduli space for the two-body system of extreme charged dilaton black holes in string theory ($a^2=1$). (a) for $N=5$, (b) for $N=4$, and (c) for $N=3$.

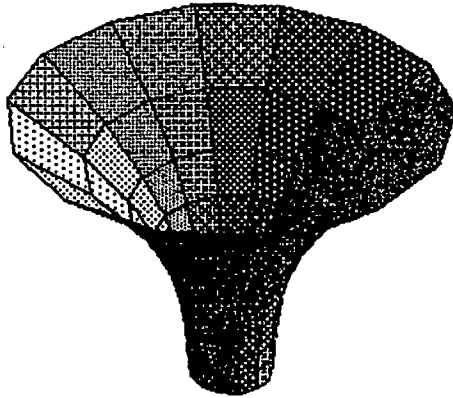
$N=3$

(a)



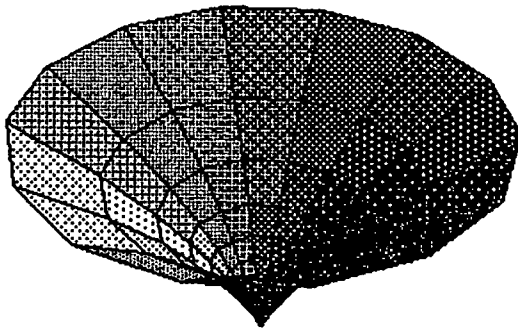
$a^2=0$

(b)



$a^2=1/3$

(c)

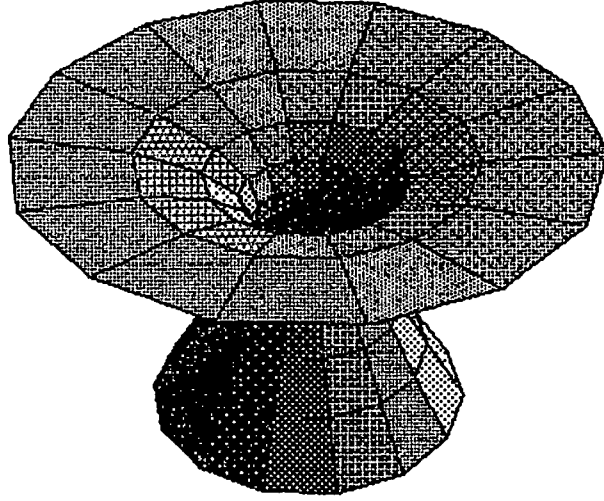


$a^2=1$

Fig. 1

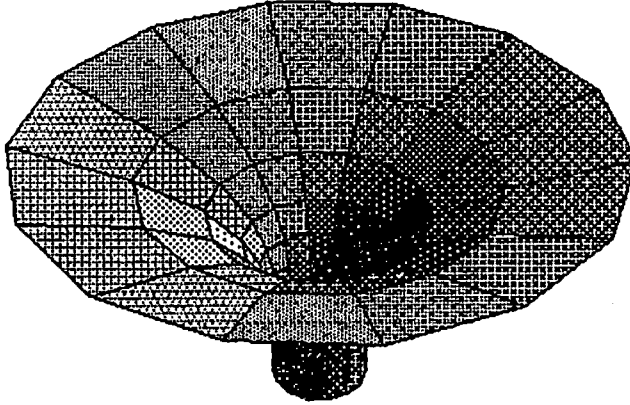
$$a^2=1$$

(a)



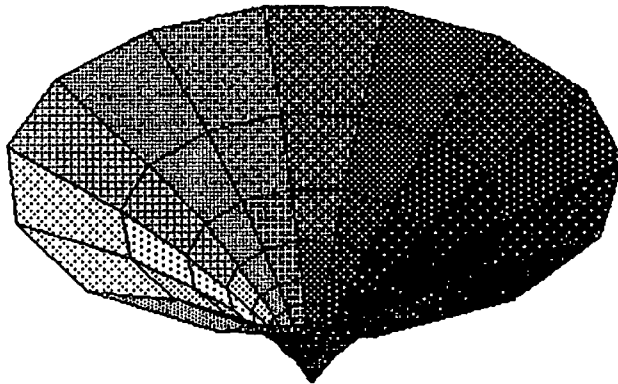
N=5

(b)



N=4

(c)



N=3

Fig. 2