

電磁場、ラグランジアン

場の古典論、ランダウ、リフシツ

$\mathcal{L}(Q, R)$

cf. Euler-Hilbert

$$Q(E, B) = \frac{1}{2} (E^2 - B^2) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$R(E, B) = \frac{1}{2} (E \cdot B)^2 = \frac{1}{32} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho}$$

cf. $F\tilde{F}$ の散度、CP 破

$$D \equiv \frac{\partial(-\mathcal{L})}{\partial Q} E + \frac{\partial(-\mathcal{L})}{\partial R} (E \cdot B) B$$

$$H \equiv \frac{\partial(-\mathcal{L})}{\partial Q} B - \frac{\partial(-\mathcal{L})}{\partial R} (E \cdot B) E$$

MAXWELL 方程式

$$\left\{ \begin{array}{l} -\frac{1}{c} D_{,t} + \nabla \times H = \mathcal{J} \\ \nabla \cdot D = \rho \end{array} \right.$$

$$= \rho$$

$$\left. \begin{array}{l} \frac{1}{c} B_{,t} + \nabla \times E = 0 \\ \nabla \cdot B = 0 \end{array} \right\} \text{ Bianchi id. } dF = 0$$

Born-Infeld-Higgs 系

における

Bogomol'nyi 方程式

について

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“電磁波”の伝わり

(各内面が“非導電体”で、空)

平面波 (線型偏光 $E \cdot B = 0$)

波面 $n \cdot x = 0 \Rightarrow n \cdot (x - vt) = 0$



$$(x' + t'v)^2 - 2(E \cdot R_x) \frac{v}{c} + [B^2 - (E_x + R_x)^2] \frac{v^2}{c^2} - t'^2 = 0$$

真空中では ($x' \neq 0$ かつ) $\frac{v}{c} = 1$

(BL) の場合

③ $B_z = B_0, R_x = R_y = 0$

$$\left(\frac{v}{c}\right)^2 = 1 - \frac{[B_0^2 - (B_0 \cdot n)^2]}{1 + R_x^2}$$

x, y 方向 $v = c \frac{1}{\sqrt{1 + R_x^2}} < c$ 光速を定より
z 方向 $v = c$ (速さが c になる)

平面波 $E = E_0 \sin, B = B_0 \sin$

$$(B^2 + E^2 - E^2) (dB^2 - dE^2) - (B \cdot dB - E \cdot dE)^2 = 0$$

$$E = nB \pm \sqrt{1 - n^2} \beta \quad \rho c n c t$$

波面 $\frac{dx}{dt} = d\omega = \frac{E \beta \pm \sqrt{1 - n^2} \beta}{1 + \beta^2} \Rightarrow n$

先角、衝撃波の発生 (shock wave)

(BL) は “例外的”

波の“非導電体”
波の“非導電体”

よって性質も、同じ

BORN-INFELD ヲクランニア PROC. R. SOC. LONDON A194
425. (1934)

$$\mathcal{L}_{BI} = \beta^2 \left[\left(1 - \frac{2G}{\rho^2} - \frac{2R}{\rho^4}\right)^{\frac{1}{2}} - 1 \right]$$

$\beta \rightarrow \infty ; d \rightarrow -Q = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \rightarrow$ (MAXWELL)

$$\mathcal{L}(BI) = \beta^2 \left[\left(1 - \frac{2Q}{\rho^2}\right)^{\frac{1}{2}} - 1 \right]$$

最初は 2次元空間 (11)

\Downarrow $D = \epsilon E, H = \frac{1}{\mu} B$

$$E = \frac{1}{\mu} = \left(1 + \frac{1}{\rho^2} (\beta^2 - E^2)\right)^{\frac{1}{2}}$$

\Downarrow Static, spherically symmetric electric field

$$D = \frac{e}{r^2} \left(\frac{r}{r_0}\right) \Rightarrow E = \frac{e}{r} \left(1 + \frac{r_0^2}{r^2}\right)^{\frac{1}{2}} \left(\frac{r}{r_0}\right)$$

$$\rho = e/r_0^2$$

$$r \rightarrow 0 \quad E_2 = \frac{e}{r_0^2} = \rho$$

$$U = \int dr \cdot r \cdot \left[\left(1 - \frac{r_0^2}{r^2}\right)^{\frac{1}{2}} - 1 \right] \sim 1 + \frac{e^2}{r_0} \text{ 有限!}$$

$$r_0 \sim \frac{e^2}{m c^2} \sim 2 \times 10^{-13}$$

古典電子半径

量子化は困難

OPEN STRING THEORY

$$S = \frac{1}{2\pi\alpha'} \left[\frac{1}{2} \int_{\Sigma} d\sigma^{\mu\nu} \partial_{\mu} X^{\rho} \partial_{\nu} X^{\rho} + i \int_{\partial\Sigma} A_{\rho} dX^{\rho} \right]$$



σ -model approach $X^{\mu} \rightarrow \bar{X}^{\mu} + \xi^{\mu}(\sigma, \tau)$

$$\frac{\delta S}{\delta X^{\mu}} + \frac{\delta S}{\delta \partial^{\alpha} X^{\mu}} + \dots \quad (\sigma\text{-model over-loops})$$

β -fn = 0 (looping + propagation) (2-loop 修正あり)

$$\beta_{\mu}^{\nu} = \partial^{\nu} F_{\mu} (1-F)^{-1}{}_{\nu} = 0$$

$$F = \lambda A$$

$$-1 \rightarrow I = \sqrt{\det(1+F)} \quad (\text{Faddeev})$$

$$\rightarrow \sqrt{\det(1+F)} (1-F)^{-1} \quad \beta_{\mu}^{\nu} = 0$$

これは正に BI の任意次元での形。
 最近 4次元 OPEN SUPERSTRING 心 finite hierarchy.
 モデルが研究されている。 2. Born & D. Dabbar
 $\text{GUT-like } SO(9) \approx SU(2) \otimes SU(2)$
 or $S_4(9)$

String 或いは他の "空想" 統一理論に於いて

4次元場の Action に 非線形 "補正" があるとする
 ・超高速化
 ・波長を比較

BI 型のものが期待されるであろう。 cf. supersym

様々なモデルを考へて

その帰結を調べたりすることが必要。

"Symmetry Breaking" は初期宇宙論

に於いても重要な関心を帯びている。

ここでは、ヒッグス-type の場を導入し、
 Symmetry breaking の付随した
 位相的欠陥の構造を調べたい。

* ヒッグス 有効理論 思い出したい *

これ 4次元 非線形 にした - あり

ヒッグス場の相互作用は

元の球面 11次元か?

くりこみ不可能

乙はじゆに

Abelian-Higgs model

$$E = \int d^3x \left[\frac{1}{4} F_{ij}^2 + (D_i \phi)^\dagger (D_i \phi) + \hat{V}(\phi) \right]$$

$\hat{V} = \hat{B} + i\hat{A}$

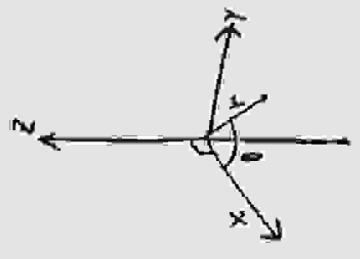
$$\begin{cases} \hat{\phi} = v\phi \\ \hat{A}_i = e v A_i \\ \hat{x}_i = (e v)^{-1} x_i \end{cases} \quad \begin{cases} \hat{V}(v) = 0 \\ \frac{\partial \hat{V}}{\partial \hat{\phi}}(v) = 0 \end{cases}$$

$$V(\vec{A}) = \frac{1}{e^2 v^2} \hat{V}(\vec{A})$$

$$E = v^2 \int d^3x \left[\frac{1}{4} F_{ij}^2 + (D_i \phi)^\dagger + V(|\phi|) \right]$$

$$V(\omega) = \frac{2V}{3\pi} \omega = 0$$

vortex configurations
ansatz $\begin{cases} \phi = f(r) e^{in\theta} \\ A_i = -\epsilon_{ij} \frac{x_j}{r^2} n(P(r)-1) \end{cases}$



$$P(0) = 1, \quad f(0) = 0$$

$$P(\infty) = 0, \quad f(\infty) = 1.$$

$$-\det(\eta_{\mu\nu} + \frac{1}{\beta} F_{\mu\nu})$$

$$= 1 + \frac{1}{\beta^2} (B^2 - E^2) - \frac{1}{\beta^4} (E \cdot B)^2$$

in 4-dim.

D次元で最高次数の項

$$\int d^4x \left[\sqrt{-\det(\eta_{\mu\nu} + \frac{1}{\beta} F_{\mu\nu})} - 1 \right]$$

$$\left(Z(\beta) = \int d^4x \right)$$

$$\beta \sim (g)^{-2} (e)$$

運動方程式

2階、連立

$$\left\{ \begin{aligned} \frac{1}{r}(r f')' &= \frac{n^2 p^2 f}{r^2} + \frac{1}{2} \frac{\partial V(r)}{\partial f} \quad \textcircled{1} \\ \left(\frac{p'}{r}\right)' &= 2 \frac{f p'}{r} \quad \textcircled{2} \quad (r' = \frac{2}{3} r) \end{aligned} \right.$$

(解析的に解けるか?)

<ここで少しおもしろいことあるか?>

$$\Delta V(f) = \frac{1}{2} (f^2 - V^2) \text{ 程度}$$

$$\left\{ \begin{aligned} f' &= \frac{n p f}{r} && \text{程度} \\ \frac{n p'}{r} &= f^2 - 1 && \text{1階} \end{aligned} \right.$$

もちろん、変数変換

$$\left\{ \begin{aligned} D_1 \phi + i D_2 \phi &= 0 \\ B &= |A|^2 - 1 \end{aligned} \right. \quad \uparrow \quad F_{xy}$$

特殊な coupling とは、大抵の場合、ポテンシャル eq. of motion
連立1階微分方程から帰着

BI Action 程度、同じようにして考えられる。

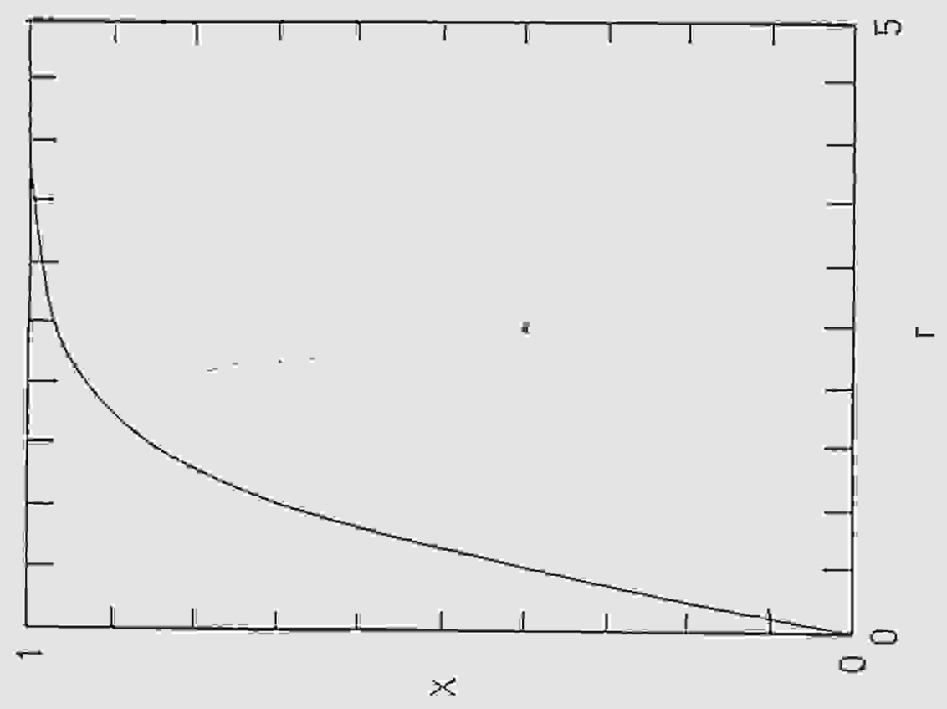


FIG.1

$$E = v^2 \int dx^3 \left[\frac{1}{2} \left(\frac{\partial \psi}{\partial t} \right)^2 + f^2 \psi + \left(\frac{\partial \psi}{\partial r} \right)^2 + \frac{1}{2} (f^2 - v^2) \psi^2 \right]$$

$$= v^2 \int dx^3 \left[\left(f^2 - \frac{v^2}{2} \right) \psi^2 + \left(\frac{\partial \psi}{\partial r} \right)^2 + \frac{1}{2} (v^2 - f^2) \psi^2 \right]$$

$$\geq 2\pi n v^2 \uparrow$$

for Vertices

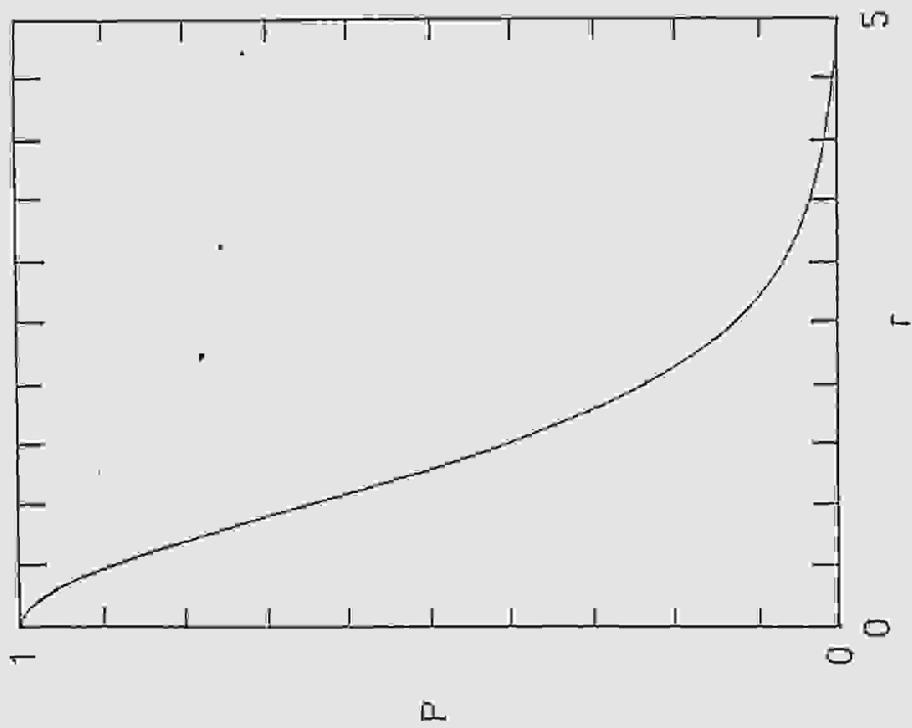


FIG.2

$$F_{\mu\nu} = \pm \tilde{F}^{\mu\nu}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma}$$

(cf. $U(1)$ \mathbb{F} \leftrightarrow density)

Bianchi id.

$$D_{[\lambda} F_{\mu\nu]} = 0$$

cf. of motion

$$D_{\mu} F^{\mu\nu} = 0$$

cf.

$$\left\{ \begin{array}{l} \delta F = 0 \\ U(1) \\ \delta \tilde{F} = 0 \end{array} \right.$$

forced, on grounds of symmetry, to say that for a direct collision (with $\pi/2$ -scattering) one half of each particle turns left while the other half turns right. In other words, the emerging particles cannot be identified with the initial particles, but rather the initial particles have broken into two halves which have reassembled differently to form the emerging particles. This picture is confirmed by consideration of the energy density as we shall now explain.

The axially symmetric 2-monopole (the "collision" state) does not look at all like a superposition of two single monopoles both sitting at the centre. On the contrary, numerical computations [18] show that the energy density has (on the (e_1, e_2) -plane) a local minimum at the centre. The maximum lies on a ring around the axis of symmetry, so that the 2-monopole is mainly concentrated in a toroidal region. The $\pi/2$ -scattering in a direct collision can then be understood in terms of the following schematic diagram of energy densities.

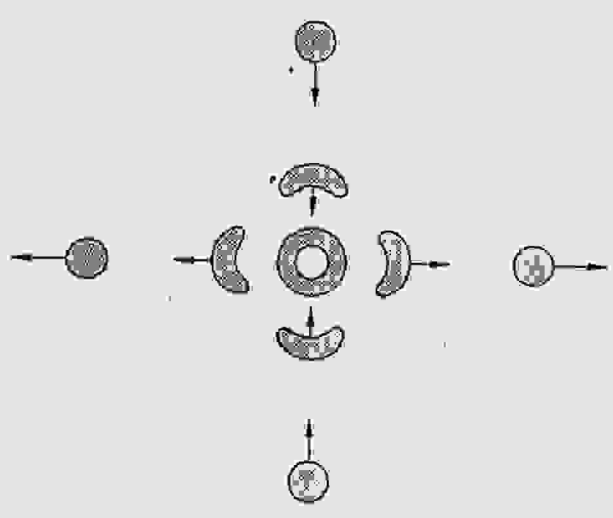


Diagram 17

The $\pi/2$ -scattering can be explained qualitatively in terms of the geometry of the surface Σ_1 . As we saw in chapter 12, Σ_1 is a convex surface of revolution, asymptotic to a cone of angle $\pi/3$. It can be viewed as a smoothed-out version of this asymptotic cone, in which the singularity at the vertex is eliminated. Now the cone represents point pairs in R^2 , i.e. it is R^2/Z_2 , and a geodesic on the cone describes free motion of non-interacting particles in the plane. In particle motion direct collision is represented by a generator of the cone; this is a geodesic which "bounces back" from the vertex. However, on any smoothed-out surface, provided circular symmetry is preserved, a geodesic orthogonal to the circles of symmetry passes through the vertex region and comes out on the opposite side as indicated in the next diagram.

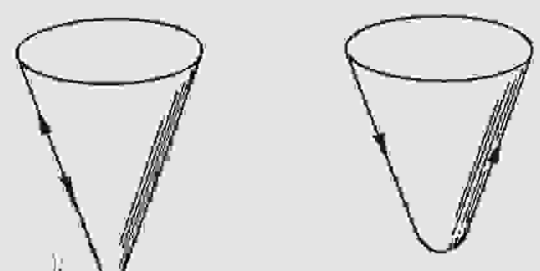


Diagram 18

Thus the $\pi/2$ -scattering of monopoles is essentially a consequence of the fact that the collision states are not singular on the configuration manifold.

We turn now to interpret geodesic motion on the surface Σ_1 . This corresponds to displacing the monopole pair in the (e_1, e_2) -plane, again with no initial electric charge. For μ large we have, as before, a small repulsive deflection. As μ decreases this becomes eventually an attraction.

Self-dual field 例子:

Cosmic String of Dimensionally-Reduced

YM Field.

内部空间 = S^2 , 半径 b .

effective in Abelian-Higgs model 在解着.

$$(ds^2 = g_{\mu\nu} dx^\mu dx^\nu + b^2 (d\theta^2 + \sin^2 \theta d\phi^2))$$

vortex 解 (重力无规)

||

(anti-)self-dual YM,

"(anti-)Instanton"

$$A_\theta = \frac{1}{2} \begin{pmatrix} i\Phi^* e^{i\theta} & 0 & -i\Phi e^{-i\theta} \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_\phi = -\frac{1}{2} \mu \sin \theta \begin{pmatrix} 0 & \Phi^* e^{i\theta} & \Phi e^{-i\theta} \\ \Phi^* e^{i\theta} & 0 & 0 \\ 0 & 0 & -(1-\cos \theta) \end{pmatrix}$$

$$A_\mu = \bar{A}_\mu(x^\mu) \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$F_{\mu\nu} = \bar{F}_{\mu\nu} \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$F_{\theta\phi} = (1-|\Phi|^2) \frac{1}{2} \mu \sin \theta \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$F_{\mu\theta} = \frac{1}{2} \begin{pmatrix} 0 & 0 & -i D_\mu \Phi e^{-i\theta} \\ i (D_\mu \Phi)^* e^{i\theta} & 0 & 0 \end{pmatrix}$$

$$F_{\mu\phi} = -\frac{1}{2} \mu \sin \theta \begin{pmatrix} 0 & 0 & D_\mu \Phi e^{-i\theta} \\ D_\mu \Phi^* e^{i\theta} & 0 & 0 \end{pmatrix}$$

$$\text{Tr } F^2 = \frac{1}{2} \bar{F}_{\mu\nu}^2 + \frac{1}{b^2} 2 |D_\mu \Phi|^2 + \frac{1}{b^2} (|\Phi|^2 - 1)^2$$

$$BI (= (BD)')$$

$$E = v^2 \int dx \left[P^2 \left(1 + \frac{1}{2P^2} F_{ij} F^{ij} \right) + (D_t \psi)^2 + V(\psi) \right]$$

eq. of motion

$$\left\{ \begin{aligned} \frac{1}{r} (r f)' &= \frac{v^2 P^2 f}{r^2} + \frac{1}{2} \frac{\partial V}{\partial f} \\ \left[\left(1 + \left(\frac{vP}{r} \right)^2 \right)^{1/2} \frac{1}{r} \right]' &= 2 \frac{f' P}{r} \end{aligned} \right. \quad \text{①} \quad \text{②}$$

$$\Phi = X(r) e^{in\psi}$$

$$\bar{A}_4 = m(P(r) - 1)$$

$$D_r \Phi = X' e^{in\psi}$$

$$D_\psi \Phi = in P X e^{in\psi}$$

$$F_{r\psi} = mP'$$

$$\text{Tr } F_{\mu\nu} F^{\mu\nu} = 4m (P(1-X'))' dr d\psi$$

$$\int \text{Tr } F_{\mu\nu} F^{\mu\nu} = -32\pi \int dr \quad \left\{ \begin{aligned} X(0) &= 0; P(0) = 1 \\ X(\infty) &= 1; P(\infty) = 0 \end{aligned} \right.$$

$$F_{\mu\nu} = -\vec{F}_{\mu\nu}$$



$$\left\{ \begin{aligned} \frac{mP'}{r} &= X^2 - 1 \\ X' &= mPX \end{aligned} \right.$$

不変関数も変化する。

$$f' = \frac{m_p f}{r} \quad (D_1 + i D_2 \phi = 0) \quad \dots \quad \text{300}$$

運動方程式に代入して、(3)を使う

$$\frac{m_p f'}{r} = \frac{1}{2} \frac{1}{f} \frac{\partial V(f)}{\partial f} \quad \text{を解く。これは①の近似線分}$$

$$V(f) = \beta^2 \left[1 - \left(1 - \frac{1}{f^2} (U^2 - V^2)^{1/2} \right)^2 \right] \quad \text{f.1.9}$$

$$\begin{cases} m_p f' \\ \text{von Neumann} \end{cases} \left\{ \begin{array}{l} f' = \frac{m_p f}{r} \\ \frac{m_p f'}{r} = \frac{1}{2} \frac{1}{f} \frac{\partial V}{\partial f} \end{array} \right.$$

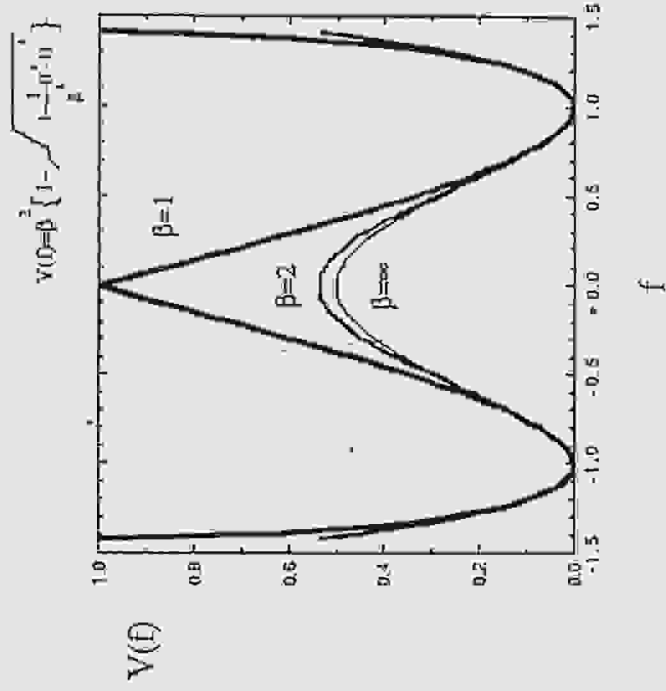
解を定めます。

Higgs の ground state の mass は小さい。

$$m_H^2 = m_A^2 = 2e^2 v^2$$

V は $f^2 - 1 < \beta$ の場合定常解が存在

$$\beta \geq 1$$



無限遠方 0 → 0.8 2.8 3.1

$$\left. \begin{aligned} f &\sim 1 - C K_0(\sqrt{2}r) \\ p &\sim C(\sqrt{2}/n) r K_1(\sqrt{2}r) \end{aligned} \right\} \beta = \infty \text{ の場合}$$

原点近傍では

$$\begin{aligned} f &= A(\sqrt{2}r)^n - A(\sqrt{2}r)^{n+2} \cdot \frac{1}{4(n+2)\sqrt{1-\frac{1}{\beta}}} + O(r^{n+4}) \\ p &= 1 - (\sqrt{2}r)^2 \cdot \frac{1}{4n\sqrt{1-\frac{1}{\beta}}} + O(r^4) \end{aligned}$$

($\beta \neq 1$)

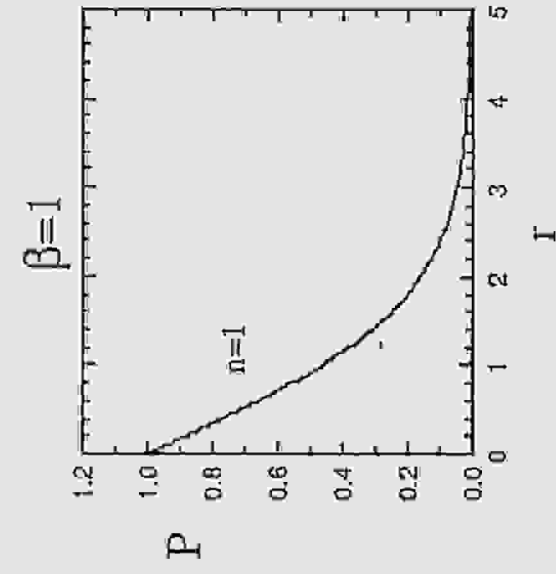
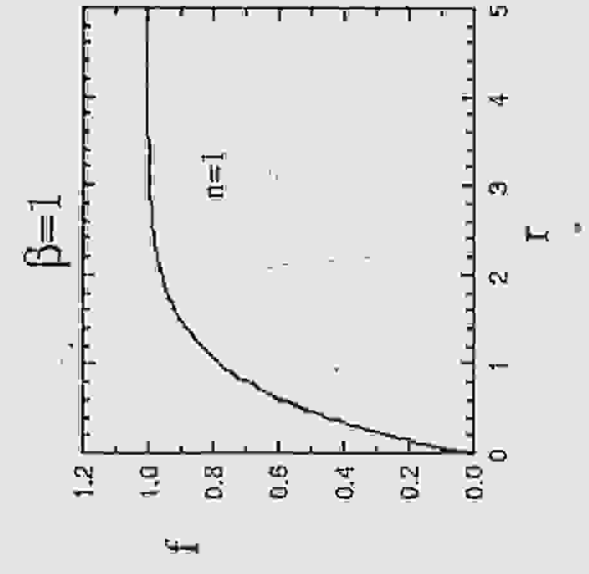
$\beta = 1$ の場合

$$\begin{aligned} f &\sim r^n \\ 1-p &\sim r^{2-n} \end{aligned}$$

$\frac{n}{\beta} = 1$ の場合 finite の場合
non-singular

$$\begin{aligned} B_z(0) &= \infty \\ H_z(r) &= \frac{n r^n}{\sqrt{1+\frac{r^2}{\beta}}} \end{aligned}$$

$$\begin{aligned} H_z(0) &= \lim_{r \rightarrow 0} \frac{\beta n r^n}{\sqrt{1+\frac{r^2}{\beta}}} \\ p &= 1, H_z(0) = n r^n < \infty \end{aligned}$$

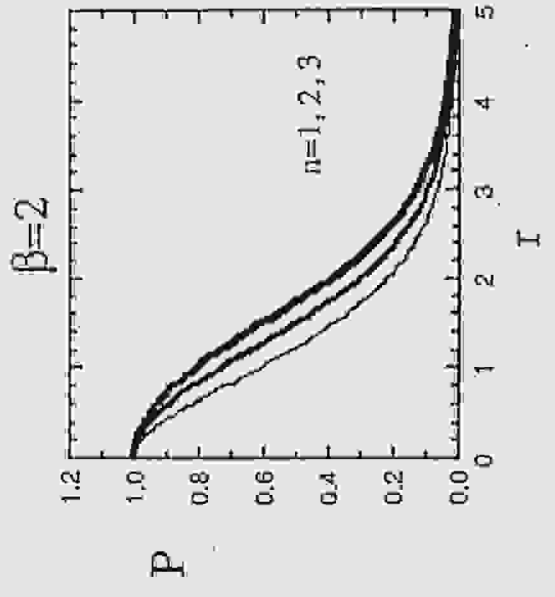
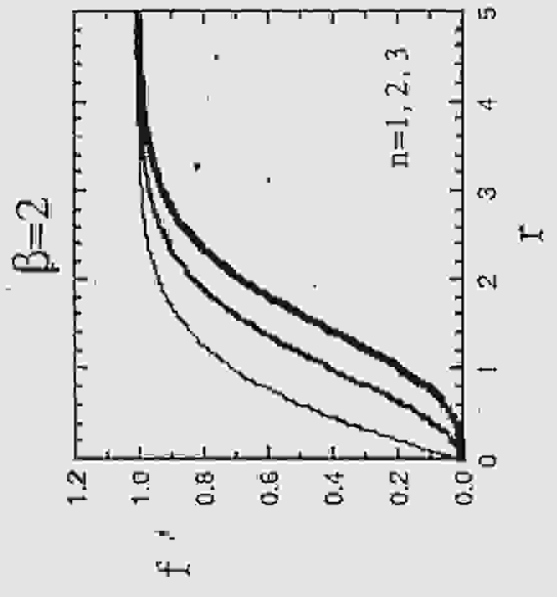


上式化简

$$\begin{aligned}
 E &= n^2 \int dx \left[\beta^2 \left\{ \left(1 + \frac{n^2 p^2}{\rho^2} \right)^{1/2} - \left(1 - \frac{1}{\rho^2} (U - U^0)^2 \right)^{1/2} \right\} + \frac{1}{2} \frac{p^2}{\rho^2} + \frac{n^2 p^2}{\rho^2} \right] \\
 &= n^2 \int dx \left[\left(1 - \frac{n^2 p^2}{\rho^2} \right)^2 + \frac{1}{2} \frac{p^2}{\rho^2} \sqrt{1 + \frac{n^2 p^2}{\rho^2}} - \sqrt{1 + \frac{n^2 p^2}{\rho^2}} \sqrt{1 - \frac{1}{\rho^2} (U - U^0)^2} \right] \\
 &\quad + \frac{1}{2} p^2 \sqrt{1 + \frac{n^2 p^2}{\rho^2}} \sqrt{1 - \frac{1}{\rho^2} (U - U^0)^2} - 1 \\
 &\quad + \frac{1}{\rho^2} \left[n^2 p^2 (U - U^0) \right]
 \end{aligned}$$

= $2\pi n v^2$ for the latter solution.

(ρ 是 z 的函数, $\rho > 0$ 是场变量的一个函数)
 量子化, 量子化可能由解 $n > 1/2 \rightarrow$ 量子化
 与反子)



② $\tau \rightarrow \tau^2$ SUPER SYMMETRY の関係
 4次元時空 τ , (BI) の形式

$$S = \int d^4x \left[\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (D_\mu \phi)^2 + \dots \right]$$

この τ -110- version の形式

Superfield V in WZ gauge

$$V = -\theta \sigma^\mu \bar{\theta} A_\mu + i \bar{\theta}^2 \theta \lambda - i \theta^2 \bar{\theta} \bar{\lambda} - \frac{1}{2} \theta^2 \bar{\theta}^2 D$$

SUSY transformation Law

$$\begin{aligned} \delta A_\mu &= i \bar{\epsilon} \gamma_\mu \epsilon \\ \delta \lambda &= -i \sigma^{\mu\nu} \epsilon \gamma_\nu A_\mu - \sigma_3 \epsilon D \\ \delta D &= -i \bar{\epsilon} \gamma_5 \epsilon \lambda \end{aligned}$$

この Lagrangian ϕ, ϕ^\dagger の必要

2つの chiral Superfield. Φ, Φ^\dagger

$\Phi^\dagger e^{2V} \Phi$... kinetic term with minimal coupling.

$$W_\alpha = -\frac{1}{4} \bar{D} \bar{D} D_\alpha V$$

$$W D V + \bar{W} \bar{D} V \supset F_{\mu\nu}^2 - 2D^2 \text{ in Dimension}$$

$$W_\alpha = D_\alpha V$$

$$W_\alpha W^\alpha + \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \supset F_{\mu\nu}^2 - 2D^2 \text{ in the first ...}$$

② non-minimal coupling

$$E = m^2 \int d^4x \left[\frac{1}{2} G(\phi) \left(1 + \frac{1}{2} F_{\mu\nu}^2 - 1 \right) + \frac{1}{2} (D\phi)^2 \right]$$

$$G(\phi) = 1 \quad \text{in } \partial G / \partial \phi = 0$$

① τ と τ^2 の関係. 運動方程式が一致する条件

$$G(\phi) = \left(1 + \frac{1}{\beta} (|\phi|^2 - 1)^2 \right)^{1/2}$$

mass $m^2 = 2e^2 v^2$

この Lagrangian-like equation is

β is a constant $\beta \rightarrow \infty$ is the limit

$$D\phi + i p_2 \phi = 0$$

$$B = |\phi|^2 - 1$$

この Lagrangian

$$E = m^2 \int d^4x \left[\frac{1}{2} \frac{G}{\sqrt{1+\frac{1}{\beta} G}} \left(\beta - \sqrt{1+\frac{1}{\beta} G} (|\phi|^2 - 1) \right)^2 + \frac{1}{2} \left(1 - \sqrt{1+\frac{1}{\beta} G} \right)^2 \right] + |D_1 \phi + i p_2 \phi|^2 + \dots$$

$= 2\pi m^2$ for vortex

$$2J_j = A_j - i \phi^\dagger D_j \phi + i \phi D_j \phi^\dagger$$

これは (7) あり (BI) と (3). (展開して)

F^2 と $2D^2$ がある. D -term 量子化.

25m. Syn. Breaking あり $V \neq 0$ を使.

(Fayet-Zel'dovich)

適当に Normalize して

$$S \sim \int d^4x \left[\beta^2 \left(\sqrt{1 + \frac{1}{\beta^2} (F^2 - D^2)} - 1 \right) + |D_\mu \phi|^2 + (|\phi|^2 - 1) \beta \right]$$

D の運動方程式を

$$S \sim \int d^4x \left[\beta^2 \left(\sqrt{1 + \frac{1}{\beta^2} (F^2 - D^2)} - 1 \right) + |D_\mu \phi|^2 \right]$$

$$D = \frac{\sqrt{1 + \frac{F^2}{\beta^2}}}{\sqrt{1 + \frac{D^2}{\beta^2}}} \quad (11-1)$$

SUSY と gauge symmetry

$$\delta \lambda = -i \sigma^{\mu\nu} \frac{1}{2} F_{\mu\nu} - \delta_5 \lambda D$$

$$D = \frac{\sqrt{1 + \frac{F^2}{\beta^2}}}{\sqrt{1 + \frac{D^2}{\beta^2}}} \quad (11-1)$$

① $\phi \equiv 0, F_{\mu\nu} \equiv 0$

$\langle D \rangle \neq 0, \langle \delta \lambda \rangle \neq 0$
SUSY broken
g. sym. unbroken

② $|\phi|^2 = 1, F_{\mu\nu} = 0$

$\langle D \rangle = 0, \langle \delta \lambda \rangle = 0$
SUSY unbroken
g. s. broken

③ vev solution

$$B_2 = (|\phi|^2 - 1)$$

$$\delta \lambda = -i \sigma^{\mu\nu} \frac{1}{2} B - \delta_5 \lambda (|\phi|^2 - 1) = (-i \sigma^{\mu\nu} - \delta_5) \frac{1}{2} B$$

$$\sigma^{\mu\nu} = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}$$

$$\delta_5 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

SUSY は "半分" broken
Majorana 4成分 $\rightarrow 3$
2成分は $\psi, \bar{\psi}$ だが $\langle \delta \lambda \rangle = 0$ となる.

B が U(1) と同じ.

Monopole & Dyon.

BI の non-Abelian への拡張.

並心 への ambiguity.

一番有名な拡張

$$\mathcal{L} = \frac{1}{2} \text{Tr} \exp \left\{ -\frac{1}{2} \text{tr} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (F_{\mu\nu})^n \right\}$$

↑
時間空間

(Atiyah & Hitchin)

ここで

$$\mathcal{L}_{BI} = \text{Tr} \rho^2 \left\{ \sqrt{1 + \frac{1}{\rho^2} (E - B)^2} - \frac{1}{\rho} (E \cdot B) - 1 \right\}$$

と (0)。

しかし

$$\sqrt{1+X} = 1 + \sum_{n=1}^{\infty} a_n(X)^n,$$

$$1 + \sum_{n=1}^{\infty} a_n X^n = \sqrt{1+X}$$

球対称な $E, B \rightarrow 0$ への Dyon の ansatz を

課したとき

は \mathcal{L}_{BI} が定数に OK.

Monopole $S(U(1)) \rightarrow U(1)$

Prasad-Sommerfeld limit

\simeq Higgs potential 0!

Bogomolnyi equations

$$B_i = D_i \phi$$

(BI) の ansatz

$$\mathcal{L} = \text{Tr} \left[\rho^2 \sqrt{1 + \frac{1}{\rho^2} (D_i \phi)^2} \sqrt{1 + \frac{1}{\rho^2} (F_{ij})^2} - 1 \right]$$

この ansatz

上の Bogomolnyi eq. を解く。

注: $\sqrt{1+(D\phi)^2}$ の type II zero-wave 解

BI と同様 場合 τ shock wave あり.

Dyon 解.

$$B_i = D_i \phi \times \omega \neq 0$$

$$E_i = D_i \phi \times \omega = 0$$

energy 0 同 ω 解

$$E = 4\pi \frac{v}{e}$$

$\phi \rightarrow 0$ (BI) への τ 解は ω の形 τ 解 ω Dyon 解 τ あり

$$dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Ansatz:

$$\frac{1}{2} \begin{pmatrix} \cos\theta \sin\theta e^{-i\tau} \\ \sin\theta e^{i\tau} - \cos\theta \end{pmatrix} \equiv \frac{1}{2} \sigma_1$$

$$\frac{1}{2} \begin{pmatrix} \sin\theta \\ -\cos\theta e^{i\tau} - \sin\theta \end{pmatrix} \equiv \frac{1}{2} \sigma_2$$

$$\frac{1}{2} \begin{pmatrix} 0 \\ ie^{i\tau} - ie^{-i\tau} \\ 0 \end{pmatrix} \equiv \frac{1}{2} \sigma_3$$

$$= (1-K) \frac{1}{2} \sigma^3$$

$$p = (1-K) \sin\theta \frac{1}{2} \sigma^2$$

B. F.

$$= \frac{H}{2r} \sigma_1$$

$$= \frac{J}{2r} \sigma_1$$

$$\cos\psi \left(\frac{H}{r}\right)' = \frac{1-K^2}{r^2}$$

$$K' = -\frac{KH}{r} \cos\psi$$

$$J = \sin\psi H$$

$$_1\phi = \left(\frac{H}{r}\right)' \frac{1}{2} \sigma_1$$

$$_0\phi = -\frac{KH}{r} \frac{1}{2} \sigma^2$$

$$_4\phi = +\frac{KH}{r} \frac{1}{2} \sigma^3 \sin\theta$$

$$F_{r\theta} = \left(\frac{J}{r}\right)' \frac{1}{2} \sigma_1$$

$$F_{\tau\theta} = +\frac{JK}{r} \frac{1}{2} \sigma^3 \sin\theta$$

$$F_{\theta 0} = -\frac{JK}{r} \frac{1}{2} \sigma^2$$

$$F_{\theta\psi} = (1-K^2) \sin\theta \frac{1}{2} \sigma_1$$

$$F_{r\psi} = -K' \frac{1}{2} \sigma^2$$

$$F_{\tau\psi} = -K' \sin\theta \frac{1}{2} \sigma^2$$

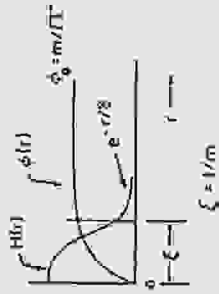


Fig. 6-7. Magnetic field strength and scalar field behavior near the vortex core.

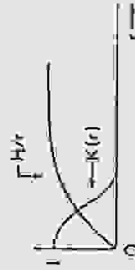


Fig. 6-9. Behavior of the functions in the Prasad-Sommerfield solution.

$$H(r) = r \coth r - 1$$

$$K(r) = \frac{r}{\sinh r}$$

はじめ

BI の Higgs の "入れ方".

("Polynomiality eqs. 失敗!")

と ZH の話

いろいろあるからこじまじま、

とこのこと)

二から...

• SUSY 及び Kaluza-Klein への関係

• 相互作用の パラメータ

例として

$$\sqrt{-det (\eta_{\mu\nu} + F_{\mu\nu} + \tilde{F}_{\mu\nu})}$$

BZ の 深さの 依存性? の話

Axionic String

BI-Gauge String の 相互作用

↑ 互換性原理

↓ Axion

• 上記 どちらか強調して

何が unique な 「粒子」 だ?