

Cosmic String

from

dimensional reduction

of

Yang-Mills system

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1. Introduction
2. Cosmic string in 4-dim.
Einstein-Yang-Mills system
3. Fermion around the string
4. Generation of strings vs. Spontaneous
Compactification in hot universe
5. Towards the construction of
Bosonic, Superconducting Cosmic String
6. Summary

Coupling to Gravity

$b \neq \text{const.}$

$$\frac{1}{\sqrt{|g|}} \mathcal{L} = \frac{1}{4\ell^2} \text{Tr} F_{\mu\nu}^2 - \frac{1}{2\kappa^2} R + \Lambda + \frac{1}{4g^2} G_{\mu\nu}^2$$

Asymptotically flat space $\otimes S^2$

U(1): $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

Monopole on S^2 $G_{\theta\phi} = -\frac{1}{2} \sin\theta$

$\Lambda = \frac{2\Lambda^2}{\kappa^2}$; $b(\infty) = \frac{\kappa^2}{4g^2}$

metric $ds^2 = e^A (-dt^2 + dz^2) + dr^2 + e^C d\psi^2$

$+ e^B (d\theta^2 + \sin^2\theta d\phi^2)$

$\left\{ \begin{array}{l} A(r) \\ C(r) \\ B(r) \end{array} \right.$

unit of length $\Rightarrow \frac{\kappa^2}{4g^2} = 1$

$B(\infty) = 0$

For $U(1)$ gauge field

$\Phi = X(r) e^{i\psi}$: $X(0) = 0, X(\infty) = 1$

$\hat{A}_\psi = P(r) - 1$: $P(0) = 1, P(\infty) = 0$

numerical calculation by computer

figures

$q^2 = \frac{e^2}{2g^2}$

$q \rightarrow \infty$ $b \sim \text{const}$
($b^2 = e^B$)

$\frac{\kappa^2}{b^2} \approx \frac{\kappa^2}{b_0^2} - 2(e^2) + \frac{1}{g^2}$

NOTE:

① $b^2 = e^B > b^2(\infty) (=1)$ from unit

$b^2(0) \sim \exp(\frac{1}{q^2})$ (ZZZ)

② deficit angle

$dr^2 + S^2 r^2 d\psi^2 = dr^2 + r^2 (d\psi')^2$

$d < 1$

$0 < \psi' < 2\pi d$



conical space

$\Delta\psi \equiv 2\pi \left(1 - \lim_{dr} \frac{d}{dr} e^{\frac{B}{2}} \right) \sim \frac{2\pi}{q^2}$

It is expected that

(Energy density per length

$\mu \propto \frac{1}{q^2}$

③ $A(r) \rightarrow$ (exponentially) flat space

near string core

attractive force

$b = e^{\frac{B}{2}}$

as a scalar boson

$ds^2 = e^{-B} \tilde{g}_{\mu\nu} dx^\mu dx^\nu + e^B d\Omega^2(S^2)$

\downarrow Let $\tilde{g}_{\mu\nu} \sim \dots + \frac{1}{2} (\tilde{g}_{\mu\nu})^2 + \frac{1}{2} \dots (1 - e^{-B})^2 + \dots$

$\tilde{T}_{\mu\nu}$ conserved! e.m. tensor

④ $q \sim 2$ critical?



cf. Kasner-like boundary

3. Fermion zero-modes

↳ fundamental rep. of $SU(2)$

Dirac operator

$$\mathcal{D} = \mathcal{D}_+ + \mathcal{D}_-$$

$$\mathcal{D}_\pm = \gamma_5 (\gamma_\mu \partial_\mu + \frac{1}{2} R)$$

($A_t = A_z = 0$ t, z → homogeneous coordinates)

of zero modes of $\mathcal{D}_\pm = \#$ z. v. of \mathcal{D}

↑
Index $I = \frac{1}{32\pi^2} \int_M \text{Tr } F \wedge F$

(r, ψ) plane
@ S^2

$$I = \int_0^{2\pi} (P(1-x^2))' dr = -1$$

for n -vortices

$$(\Phi = X e^{in\psi} \quad \tilde{A}_\psi = n(P-1) \text{ etc.})$$

$$I = -n$$

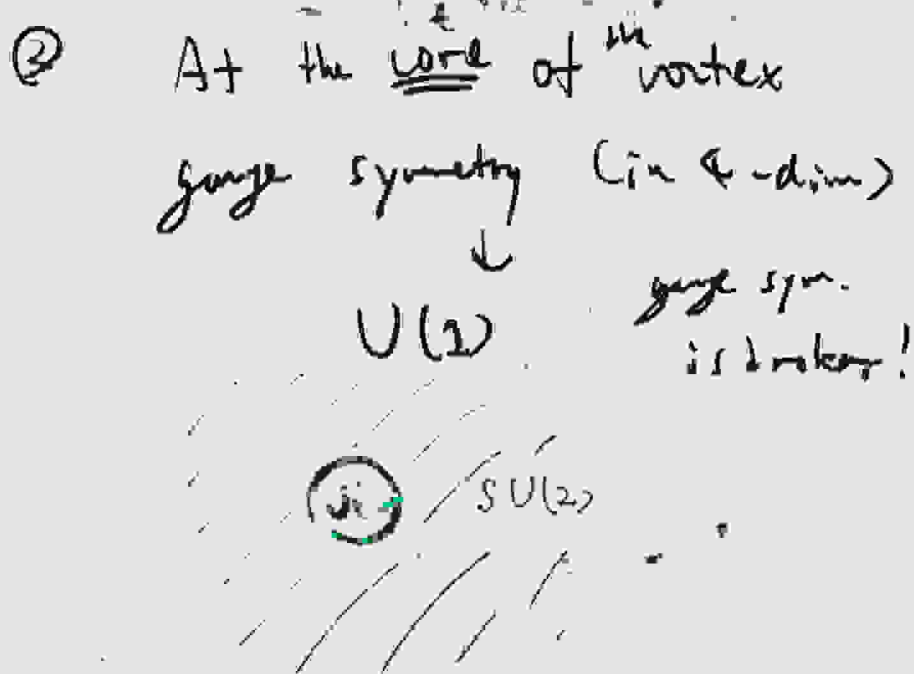
(NOTE: When coupled to gravity

the index never changes

because $R \wedge R = 0$)

AT LEAST $|n|$ -zero modes

NOTE: ① critical relation between
 "Higgs" self coupling and "U(1)" gauge
 coupling ... (where they are implicit)
 (Type I/II superconductor)



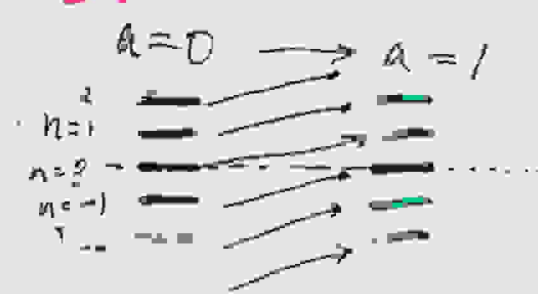
o contradiction?
 AH model \rightarrow $U(1)$ is broken in true vacuum
 $\tilde{A}_\mu \rightarrow$ massive!

YES.
 BUT often Kaluza-Klein modes
 of YM field
 become massive vector bosons.

cf. 5-dim $SU(2)$ gauge theory.

$\langle A_5 \rangle \propto a \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$(\text{mass})^2 \propto (n+a)^2$
 $A_\mu^i A_\mu^i$
 $n: \text{integer}$



o Summary

o to be calculated



OUR Cosmic string is

o "deformed" instanton.

o $G_{\mu} \sim \frac{1}{q^2} \times O(1, 1) \sim \frac{1}{(l_p^2)^2} \frac{(l_p^2)^2}{b^2(\infty)}$
 $\sim 10^{-67}$

o BSCCS * decoupling to $U(1)$

↓
 $U(1)$ magnetic field
parallel to string

$U(1) \Rightarrow$ "support" term? ?
or \neq ?

o other gauge group on various manifold

o Non Linear Sigma model.