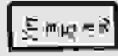


Non-Abelian Gauge Field Wormholes

in Diverse dimensions



S. Hirabayashi (TMU)

& K. S. (UMS)

1. 序
2. 2次元 \rightarrow NAGF with
3. 高次元
4. 4次元
5. 3次元 (Chern-Simons \rightarrow etc)
6. まとめ

'90 1/17

1. LITERATURE REVIEW on U.S.

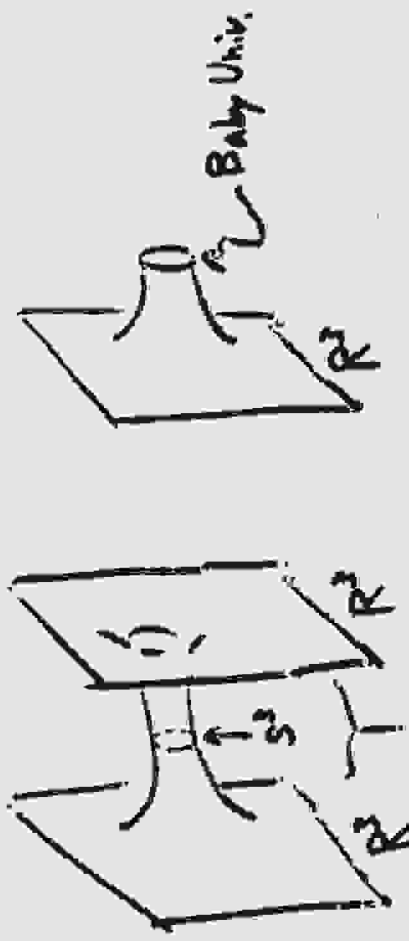
NON-TRIVIAL TOPOLOGY OF SPACE-TIME



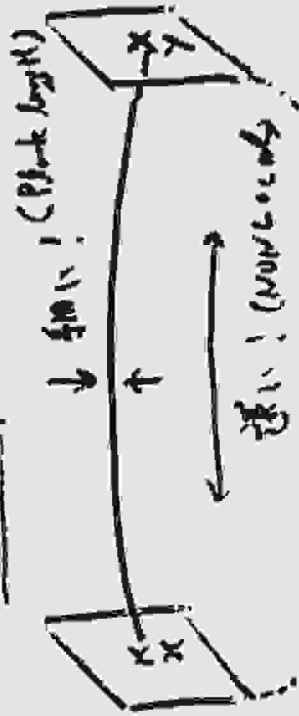
Path Integral over Euclidean Spacetime

manifolds

(Hawking's Approach to Quantum Gravity)



Wormhole



effective action

$$S \rightarrow S + \int d^4x K(x) \psi(x) + \int d^4y K(y) \psi(y)$$

cf. A. Linde

$$K(x) = K_0(x) + \dots \rightarrow V$$

$$e^{-S-V^2} \sim \int dx e^{-ikx} e^{-S-Vx}$$

cosmological unit.

$$\text{bare } \Lambda \rightarrow \Lambda + k_0 a$$

= 0 以上に含った結合 "定数" の Witten 2
"randomize" した。

宇常項 $\rightarrow 0$!
 $\left\{ \begin{array}{l} \theta \rightarrow 1 \rightarrow 0 \\ \theta \rightarrow 0 \rightarrow 0 \end{array} \right. !?$

怪しい区

- dilution 近似
- large wk.
- phase of peak integrated
- Endlicher formalism

三全発作
Derivatives

具体的な解

理論, 武器庫, 準備

- Anion
- Scalar
- Gauge field ... YM (Holographic QCD)
- higher-dimensional gravity (Martin - Davidson)

2. $Z^2 \times S^1 \rightarrow U(1) \times U(1)$

$$I = \int dx \sqrt{g} \left[-\frac{1}{2} R + \frac{1}{2} (H_{\mu\nu})^2 \right]$$

$$dS^4 = dt^2 + A^2(t) d\Omega^3(S^3)$$

$$H_{\mu\nu} \sim C \epsilon_{ij} e^i e^j$$

$$\text{Bianchi: } \epsilon^{\mu\nu\lambda\rho} \partial_\mu H_{\nu\sigma} = 0$$

$$\text{Duality: } \epsilon^{\mu\nu\lambda\rho} H_{\nu\sigma} \equiv \partial^\mu \phi$$

$\rightarrow \square \phi = 0$ massless (quadratic) scalar

$$\left(\frac{\dot{A}}{A}\right)^2 = \frac{1}{a^2} - \frac{m^2 c^2}{a^6}$$

$$\tau = 0: a = \sqrt{m c}$$

$$\tau \rightarrow \infty: a \sim \tau$$



$$\text{scalar } \tau \sim \sqrt{A} \quad \square \phi = \frac{1}{a^2} (\dot{a}^2 + \dot{\phi}^2) = 0$$

$\phi = \frac{c}{a^3}$ 運動方程式は $\Lambda \dot{a}^2 + \dot{\phi}^2$ 注意!
 \uparrow imaginary

2009 - via potentials

Wine-bottle Ψ^*
 \rightarrow WH with Finite Action
 monotonous (Goldstone) mode



$\phi \sim f e^{i\psi}$ ψ : cyclic
 coordinate



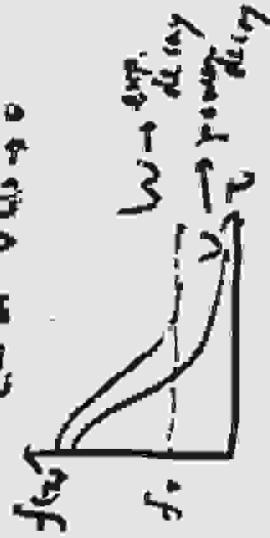
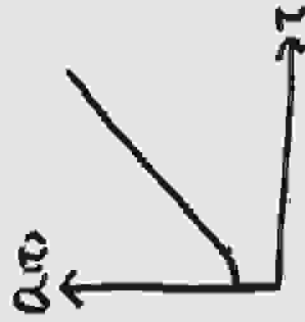
particle Ψ^*
 \rightarrow WH with INFINITE ACTION

運動方程式

$$\ddot{f} + 3\frac{\dot{a}}{a}\dot{f} = V'(f) - \frac{m^2}{4\pi^2 f^2 a^6}$$

$$\dot{a}^2 = 1 - \frac{m^2}{3} a^2 \left[V(f) + \frac{m^2}{2\pi^2 f^2 a^6} - \frac{1}{2} \dot{f}^2 \right]$$

WH \rightarrow $\partial \tau \rightarrow \tau$ 固定
 $\tau \rightarrow \infty \Rightarrow V(f) \rightarrow 0$



*Abbott-Live + K. Lee + Coleman-Lee

Gauge field in Coset space



Self-interacting scalar

$\therefore \tau$ は wine-bottle Ψ potential
 $\tau \rightarrow$ 場合 τ 固定.

(10) $SU(d+1)$ on $S^d = \frac{SO(d+1)}{SO(d)}$

$$A = A_n e^n = A_i e^i + A_n e^n$$

$i=1, \dots, d; n=1, \dots, \frac{1}{2}d(d-1)$

$$F = dA + \frac{1}{2}i A \wedge A$$

$$de^n = \frac{1}{2}dA^m e^m \wedge e^n + \dots$$

$$[A_n, A_m] = i f^{nm} A_l$$

$$\left(\begin{array}{c} 1 \\ \vdots \\ \vdots \\ 0 \end{array} \right) (A_i)_{nf} = \delta_{n,1+i} \delta_{f,i} \cdot \chi - \delta_{n,i} \delta_{f,1+i} \chi^2$$

$$Tr F \wedge F \wedge \dots = 4d |\chi|^2 + 2d(d-1) \chi \wedge \chi$$

Volkovikov - Kuybisov - Mourou

一般 π ($\pi \neq \tau$)

$\eta \approx 7$

特徴

$A(\infty) \sim X \sim \tau$
 $f(x \rightarrow \infty) \sim 1 + O(\tau^{-\delta})$
 $\delta = \frac{d-3 + \sqrt{(d-3)^2 + 8A_0^2}}{2} > d-3$ power decay
 (a/p=1) $\rightarrow \tau \rightarrow d=9$ if $f \rightarrow 2$

$a/p=1$
 $d=5 : f=4$
 $d=9 : f=8$

いづれも Action は FINITE
 ($+5 \leq f \rightarrow 1$)

$f \sim 1000$ 程度 $\tau \rightarrow f \sim 1 \rightarrow \text{short}$

$S \sim A_0^{d-1} \sim \frac{d-1}{2\pi^2} A_0^{d-1}$
 (for $\tau \rightarrow \infty$) $\sim \frac{d-1}{2\pi^2} A_0^{d-1}$

$X \rightarrow \infty$ $f \rightarrow 1$... gauge symmetry is
 asymptotic to SE state ZAKHAROV
 $f \rightarrow \infty$ $A_0 \rightarrow \infty$ (cf. NOVD)
 $f \rightarrow \infty$ $f \rightarrow \infty$ (cf. A-W)

一般 k

$\text{Tr } F^2 = 4d \cdot q \cdot |\bar{X}|^2 + 2d(d-2) \overline{(1-|X|^2)^2}$

on S^d with unit radius.

Action $(d+1) \text{ dim.}$

$\int_{S^d} d\Omega \sqrt{g} \left[-\frac{1}{2\pi^2} R + \frac{1}{4\pi^2} \text{Tr } F^2 \right]$

$dS^2 = d\tau^2 + R^2(\tau) d\Omega^2(S^d)$

$\chi = f e^{i\chi}$

$k^2 = \frac{N^2}{\epsilon^2} \frac{q}{(d-2)^2} \frac{q}{\rho} : \tau = kX$

$L^2 = \frac{1}{k^2} \frac{q}{\rho} : a = kA$

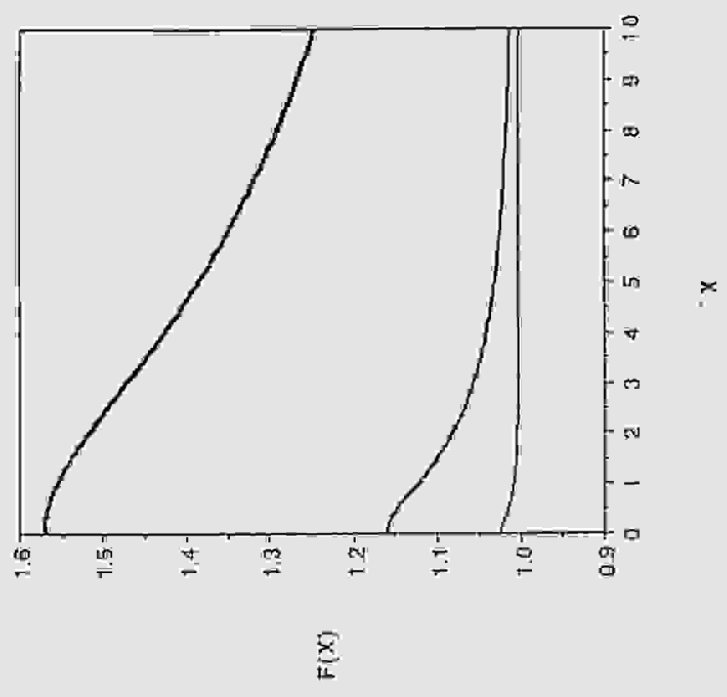
$A^{d-2} f^2 \psi' = i \sqrt{2g} L$

$f'' + (d-2) \frac{A'}{A} f' = \frac{1}{L^2} \frac{1}{A^2} f(U^2) - \frac{2qL^2}{\int^3 A^{d-2} q}$

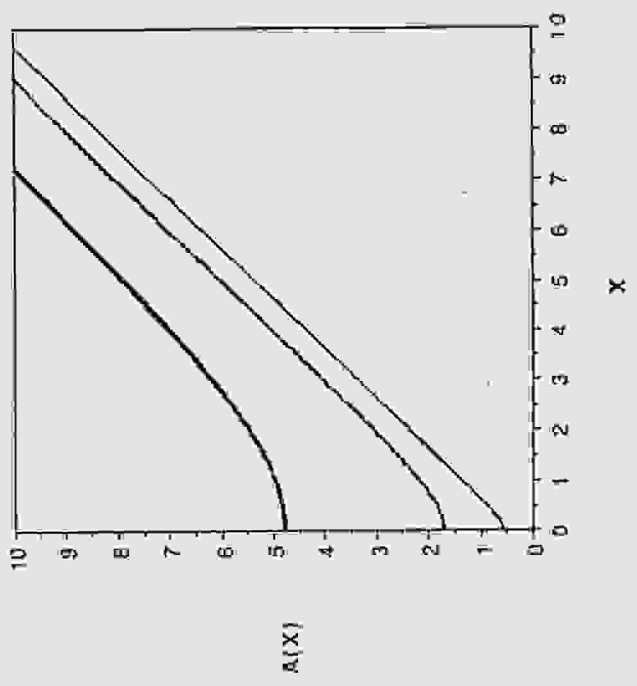
$A'^2 = 1 - A^2 \left[\frac{1}{4L^2} \frac{(U^2)'}{A^2} + \frac{q}{\int^3 A^{d-2} q} - \frac{1}{2L^2} \frac{f'}{A} \right]$

$d \geq 4, d=3, d=2$

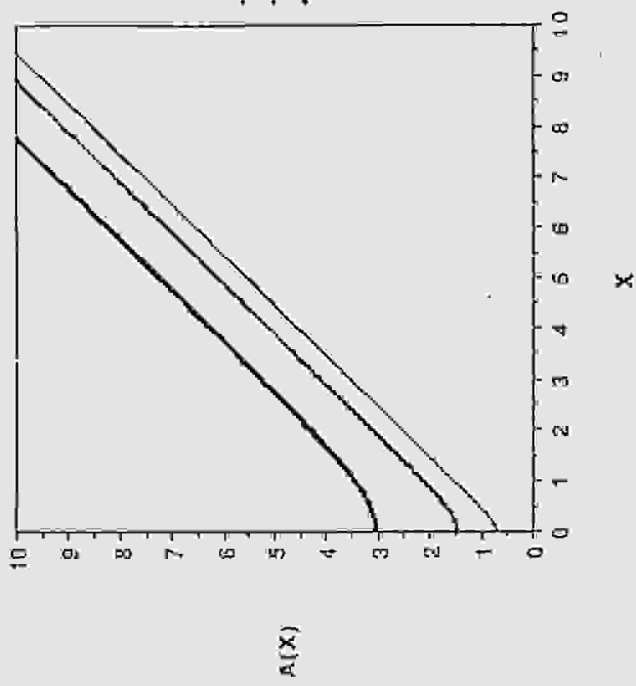
d=4 case



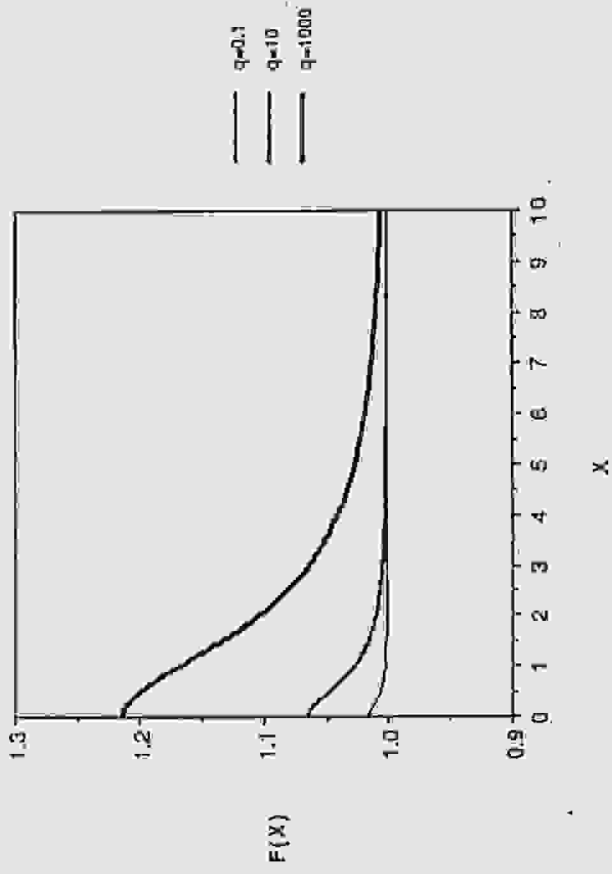
d=4 case



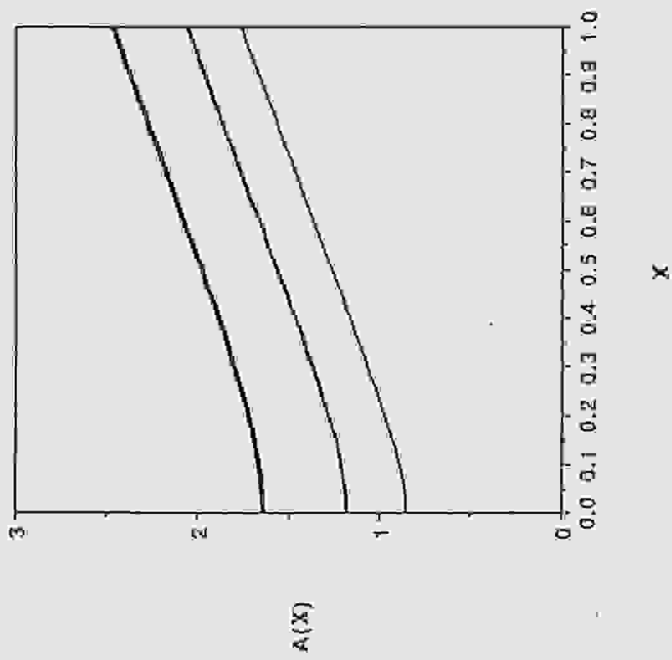
d=5 case



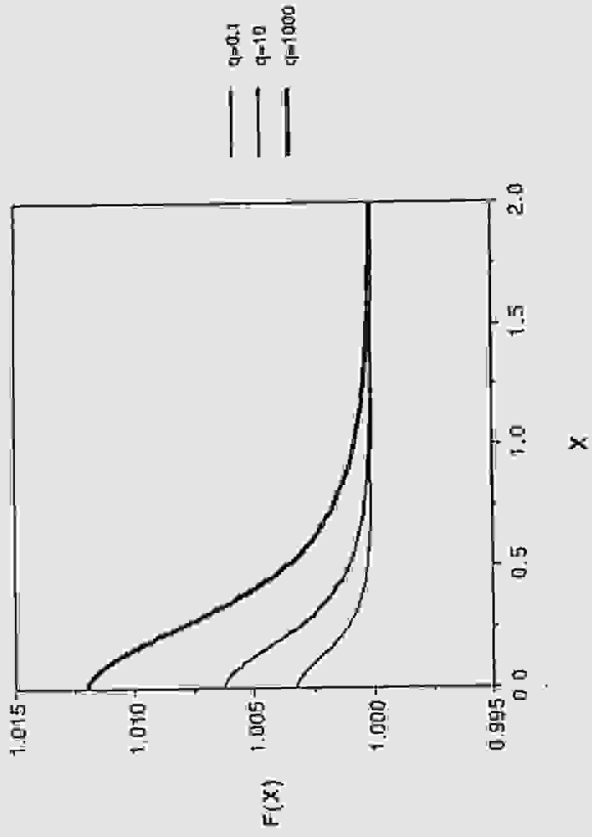
d=5 case

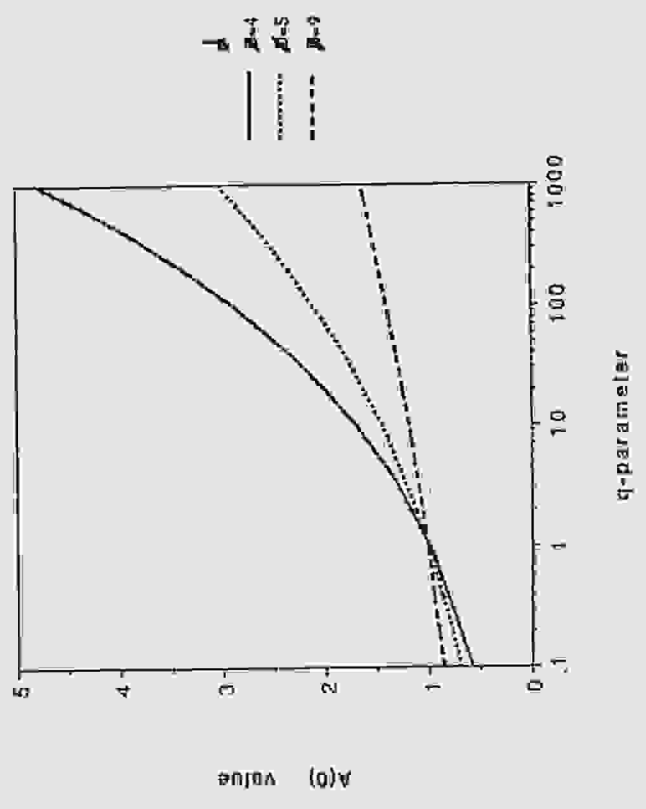
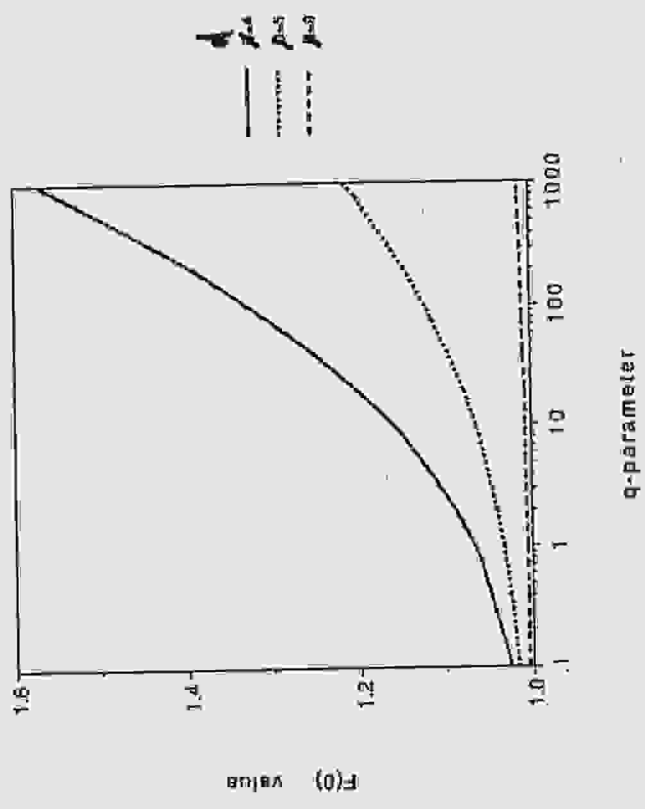


d=9 case



d=9 case

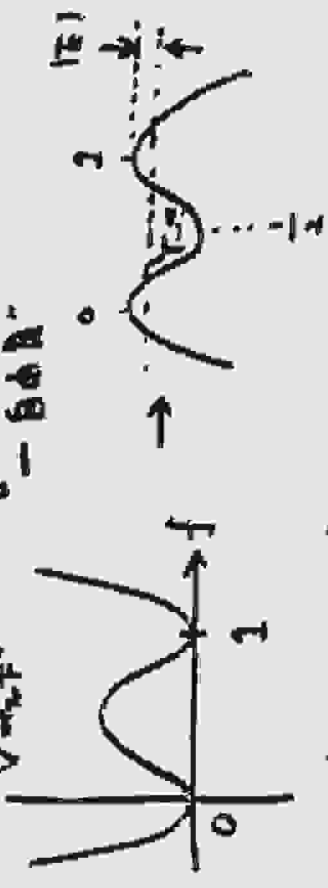




Hosoya - Ojima - Vachin - Poulkoo

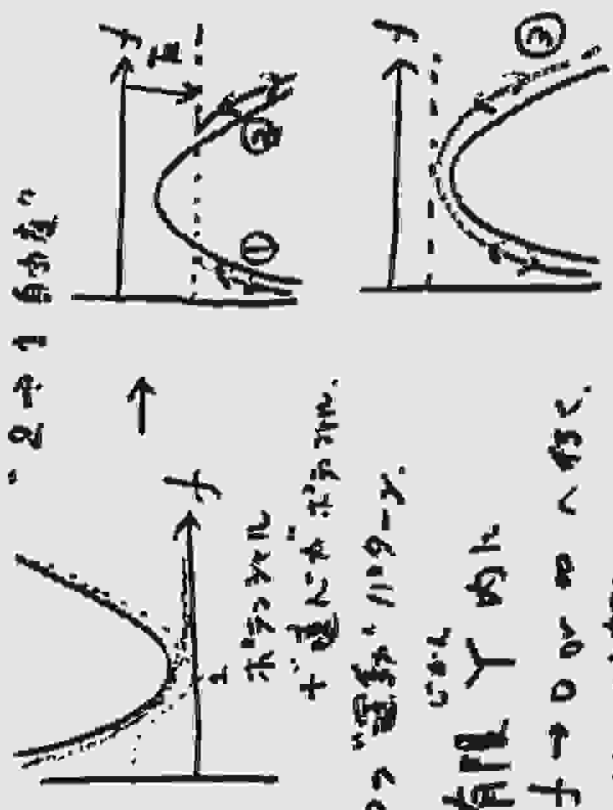
SU(2) on S³

V_{min} = "自由A"



相対的場合 SU(4) on S³

"2 → 1 自由度"



3 → "運動" の自由度
+ 運動の自由度

有限 Y の内

f → 0 or ∞ へ行く

Action の 3 次元 Lagrangian density を 求め!

with cut off or Λ⁻¹ は + 分小 e.c. の 1/Λ の 存在

d = 3 4次元

$$A'^2 = 1 - A^2 \left[\frac{1}{4L^2} \frac{(f^2-1)^2}{A^4} + \frac{f}{f^2 A^2} - \frac{1}{2L^2} \frac{f''}{A^2} \right] - \frac{\Lambda^2}{3} A^2$$

↑
c.c.

$$f'' + A' f' = \frac{1}{L^2} f U' - \frac{2fL^2}{f^2 A^2}$$

$$L^2 = \frac{1}{2} \frac{f}{f'}$$

積分して

$$(A' f')^2 = \frac{1}{2L^2} (f^2-1)^2 + \frac{2fL^2}{f^2} - 2L^2 E$$

↑
積分定数

$$A'^2 = 1 - \frac{E}{A^2} - \frac{\Lambda^2}{3} A^2$$

$$dx = A dy \quad \text{or } Y \text{ の 2次元}$$

$$Y(\rho) - Y_0 = \int_0^\rho \frac{df}{\sqrt{\frac{1}{2L^2} (f^2-1)^2 + \frac{2fL^2}{f^2} - 2EL^2}}$$

and if Λ = 0 $A = \sqrt{E} \cosh Y$

$a=2$ 球壳

SU(2) on S^2 minimal model!

$$A_0 = \frac{1}{2} \begin{pmatrix} 0 & 0 \\ i g^2 \theta^{14} & -i g^2 \theta^{14} \end{pmatrix}$$

$$A_4 = -\frac{1}{2} R^{-1} \begin{pmatrix} 0 & 0 \\ i g^2 \theta^{14} & 2e^{-i\theta} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Tr F^2 = \frac{2}{R^2} |\dot{\theta}|^2 + \frac{1}{R^2} (1 - H(\theta))^2$$

$$\underline{Q} = f e^{i\theta}$$

Action

$$I = \int d^2x \left[\frac{1}{2\alpha^2} R + \frac{1}{4\alpha^2} R^2 \right] + \frac{\mu^2}{2} \int d^2x \left[F^2 - \frac{1}{2} R^2 \right]$$

$$\mu^2 = i\mu$$

$$\epsilon_F Tr (F^2 A^2 - \dots) = i (\dot{\theta}^2 - \theta^2)$$

$$\frac{1}{\alpha^2} \dot{\theta}^2 = i\mu \dot{\theta}^2 + i\mu$$

$$\ddot{\theta} = \frac{\mu^2}{4\alpha^2} (1 - \theta^2); \quad \dot{\theta} = 0 \Rightarrow \theta = \pm \frac{\pi}{2} \Rightarrow \frac{1}{R^2} = \frac{1}{\alpha^2}$$

$$\frac{\mu^2}{4\alpha^2} |\dot{\theta}|^2 \rightarrow 2; \quad \left| \frac{\mu^2}{4\alpha^2} \right| \dot{\theta}^2 = F^2 \quad \left| \frac{\mu^2}{4\alpha^2} \right| = m < 1$$

$$\ddot{\theta} = \frac{1}{4m} \frac{1}{R^2} (1 - |\dot{\theta}|^2)^2 \quad \epsilon = \mu^2 \theta^2$$

$$\ddot{\theta} = \epsilon F (1 - \frac{1}{R^2}) + \frac{1}{R^2} F (m F^2 - 1)$$

HODD f, a



振动的解
cut off the curve
finite to Action on S^2

cut off \mathbb{Q} case

$E \gg \mu$



$$f^2 \approx \frac{\sqrt{E}}{\sqrt{E} L E}$$

Faber 2.2

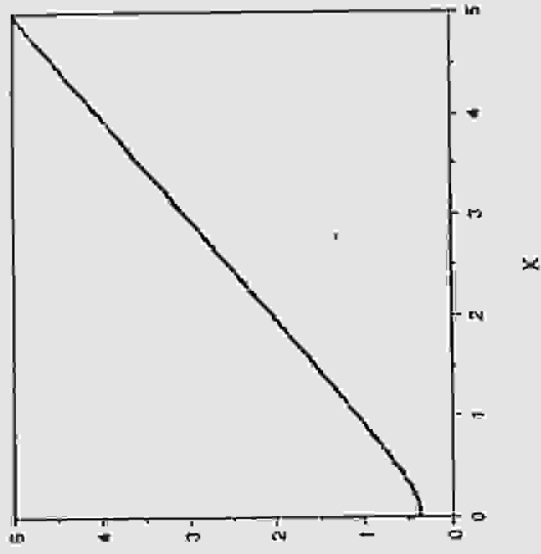
$$\therefore f \rightarrow m \rightarrow \sqrt{|f|} \quad \text{jump sym. breaker}$$

$$1 > f(0) > \sqrt{|f|}$$

↓
write out $\varphi \circ \eta$

$$\varphi \rightarrow x \quad f(x) \rightarrow z$$

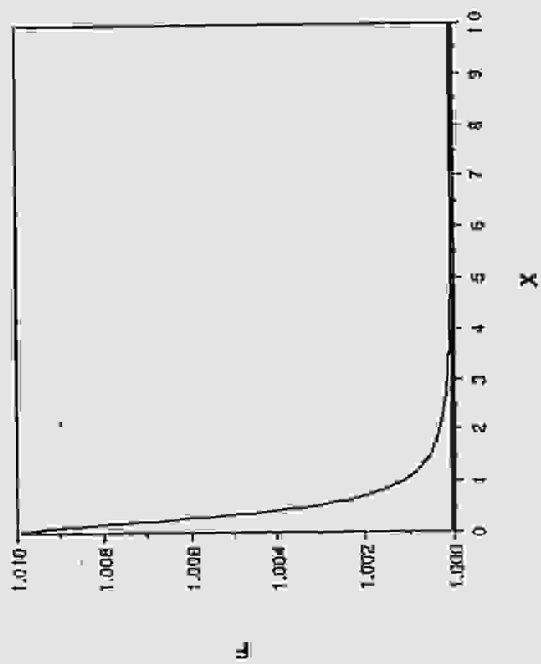
M=0.5 eps=1d2 & 1d5



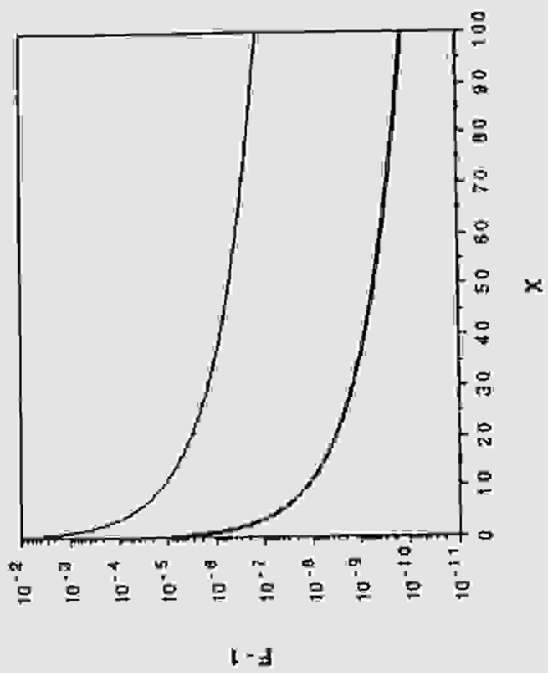
A=eps=1d2
A=eps=1d5

4

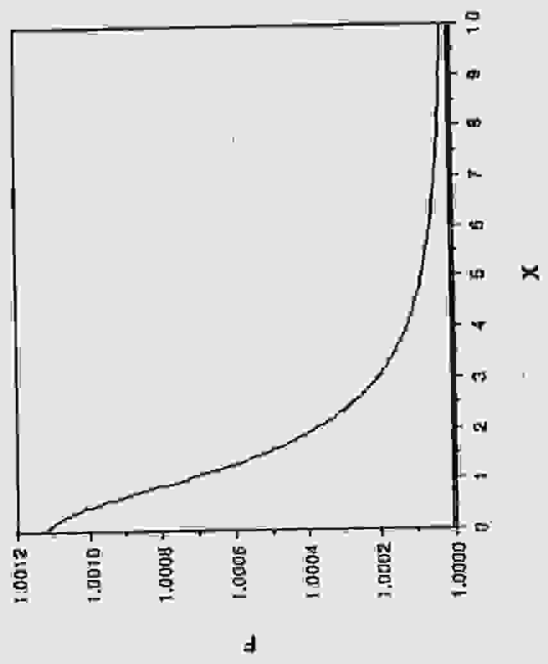
M=0.5 eps=1d2 & 1d5



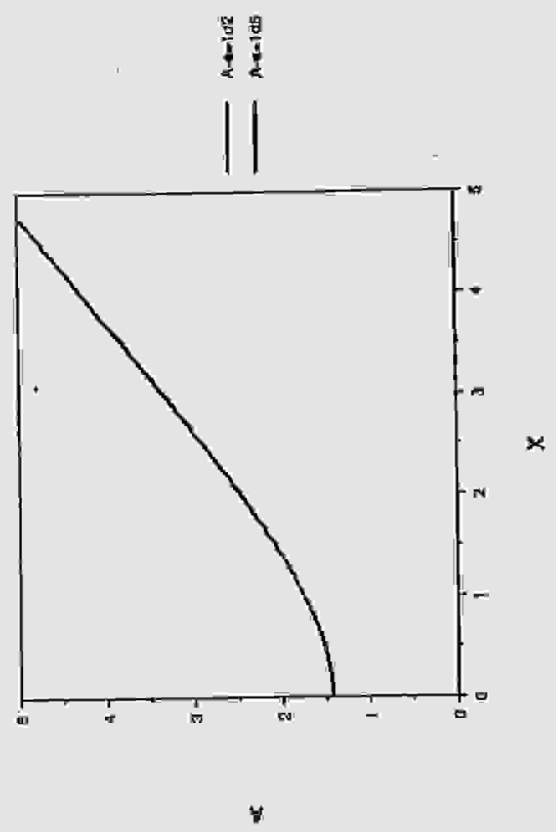
M=0.5 eps=1d2 & 1d5



M=0.1 eps=1d2 & 1d5



M=0.1 eps=1d2 & 1d5



$$m = \frac{\rho}{\rho_0}$$

$$c = e^{\beta u} \rho^2$$

370.

① 高次元 ϵ^{11}

漸近理論は有限作用積分
 χ による解が存在. Gauge sym. 破れ.
 A-W, 2nd-0 場と χ 類似.
 non power decay

- ② $d=2, 3$ gauge sym. 破れ大解.
- $d=3$ cutoff in 有限 Action

$\eta^{\mu\nu}$ coupling は 4次元で dimensionless.
 質量. 値次元では 宇宙論的尺度で

WH は CS theory 等

WH dynamics vs gravity?

②' 次元は ± 5 に "次元" $\eta^{\mu\nu}$ 群
 \rightarrow Non-Abelian sector boson
 cf. T. Okada, et al.

