

## §1. Motivation & Scenario

(Unified) Theories in Higher-Dimensional Space-Time

Kaluza-Klein (Super) Gravity

(Super) String Theories?

need COMPACTIFICATIONS

The Universe as a Topological Defect

in a Higher-Dimensional Einstein-Yang-Mills Theory

Rubakov & Shaposhnikov (1983)

"Do we live in a Domain Wall?"

Phys. Lett. 125B, 136

$$\mathcal{L} = \frac{1}{2} (\partial \varphi)^2 - \frac{1}{2} m^2 \varphi^2 + \frac{1}{4} \lambda \varphi^4$$

$$\varphi_{\text{Kink}}(y) = \frac{m}{\sqrt{\lambda}} \tanh\left(\frac{m}{\sqrt{2}} y\right)$$



fermion with Yukawa coupling  
 $i \not{\partial} \psi + h \varphi \psi = 0$   
 ↑ kink

Svetovoi & Khariton (1988)  
 (Abelian Higgs model in 6 dimensions.)

Sov. J. Nucl. Phys. 48, 250

possibilities of interest.

Phase transition = Dimensional Reduction  
 In the very early Universe  
 compatible to INFLATION?

Many Worlds? (Lense, 1911)



Worlds in Collision

ORIGIN of SCALAR FIELDS?  
 (Elementary??)

with mass of the order of Planck scale  
 and couplings of appropriate strength.

Couple to GRAVITY

(or origin of gravity)

Ref: 1911

Kubiyshin, Mironov & Volobuyev  
 Phys. Lett. 203B (1988) 379  
 and references therein

ex. on  $CP^m \times SU(m+1)$   
 $\downarrow$   
 Abelian Higgs model.

§3. Solutions coupled with GRAVITY  
 Einstein-Yang-Mills

$$\mathcal{L} \sim -\frac{1}{2\kappa^2} R + \frac{1}{4g^2} \text{tr} F_{\mu\nu} F^{\mu\nu} + \Lambda$$

$$\left( + \frac{1}{4} (R^2 - 4R^{\prime 2} + R^{\prime\prime 2}) \right.$$

$$+ \frac{1}{4g^2} G_{\mu\nu} G^{\mu\nu}$$

$$\left. + \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} + \dots \right)$$

Scenario  
 for example

$$M^8$$

by hand at first



induced by UV size shells of  $G_{\text{grav}}$

$$M^6 \otimes S^2$$



topological extended object  
 ("strings")  
 ("vortices")

$$M^4 \otimes S^1 \otimes S^1$$

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kink solution.

$$dS^2 = h^2(d\tau)^2 + a^2(z)dz^2 + b^2(z)d\Omega^2(S^2)$$

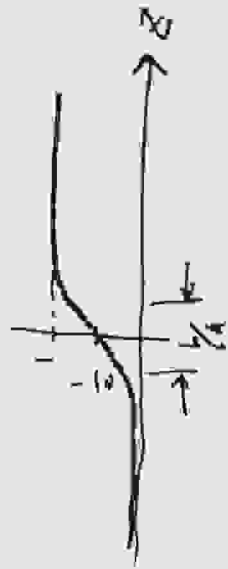
Y-M eq.  $D_\mu F^\mu = 0$

$$\left(\frac{b}{a} f'\right)' = 4 \frac{a}{b} (f-f^3)(1-2f)$$

if  $a(z) \propto b(z)$

kink solution

$$f(z) = \frac{1}{2} (1 + \tanh(\frac{a}{b} z))$$



$$f=0 \quad f=1$$



Domain Wall.

NEXT coupled to Einstein eq.

solution.

$$a(z) = \frac{1}{\cosh(\frac{z}{B})}$$

$$b(z) = \frac{B}{\cosh(\frac{z}{B})}$$

with  $\frac{3}{B^2} = \frac{h^2}{\Lambda^2} \left[ \frac{3}{2R^2} \frac{1}{B^4} + 2\Lambda \right]$

$$R_{\mu\nu}^{(d)} = \frac{h^2}{\Lambda^2} \left[ 2\Lambda - \frac{3}{2} \frac{1}{B^4} \right] g_{\mu\nu}$$

$$\frac{1}{\cosh^2 \frac{z}{B}} (d\tau^2 + B^2 d\Omega^2(S^3))$$

$$\Downarrow \quad \frac{r_c}{c} = \text{exp}\left(\frac{z}{B}\right)$$

$$\frac{4(C^2 B^2)}{(r^2 + C^2)^2} (dr^2 + r^2 d\Omega^2(S^2)) \Rightarrow dS^2(S^4)$$

$$f = \frac{r^2}{C^2 + r^2}$$

$$\Phi = |\Phi| e^{i\psi} \quad \text{"vortex" ansatz}$$

$$ds^2 = dr^2 (N^2) + g^2(r) dp^2 + a^2(r) dy^2 + b^2(r) (d\theta^2 + r^2 d\phi^2)$$

solution:

$$|\Phi| (p) = \frac{p/\beta}{\sqrt{1 + (p/\beta)^2}} \quad B^2 = \frac{h^2}{2e^2}$$

$$A + (p) = \frac{1}{\sqrt{1 + (p/\beta)^2}} - 1$$

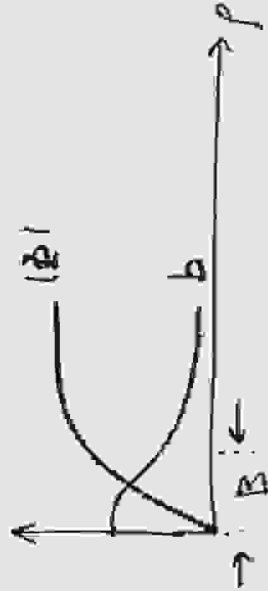
$$g^2(p) = \frac{1}{(1 + (p/\beta)^2)^2}$$

$$a^2(p) = \frac{p^2}{1 + (p/\beta)^2}$$

$$b^2(p) = \frac{B^2}{1 + (p/\beta)^2}$$

$$\text{if } \Lambda = \frac{6e^2}{h^2}$$

$h^2$  is that



↓

$$ds^2 = dr^2 (N^2) + \beta^2 dr^2 + r^2 d\psi^2 + \omega^2 r^2 (d\theta^2 + r^2 d\phi^2)$$

↳ This is  $S^4$ !

$$X_1 = \text{wir wagt}$$

$$X_2 = \text{wir wir}$$

$$X_3 = \text{wir mit wagt}$$

$$X_4 = \text{wir mit wir}$$

$$X_5 = \text{wir wagt}$$

$$X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 = 1$$

$$ds^2(S^4) = dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2 + dX_5^2 = dr^2 + r^2 d\psi^2 + \omega^2 r^2 (d\theta^2 + r^2 d\phi^2)$$

### COMPACTIFICATION

LITERALLY

The solution is equivalent to

the one offered by

S. Randjbar-Paeni, A. Salam & J. Strathdee

"Instanton-Induced Compactification"

§ 4. Phase transition

if an effective potential looks like



at  $T \neq 0$



at  $T \approx 0$

as Higgs pot.

at bottom  
 $|\Phi|=1$

$A_m = 0$  (mod. gauge tr.)

at top  
 $\Phi=0$

$A_\Phi = \frac{1}{2} \begin{pmatrix} 1-\cos\theta & 0 \\ 0 & -1-\cos\theta \end{pmatrix}$

"MONOPOLE" Config.

↓  
fermion-zero modes

ONE LOOP

Free Energy of Fermions

coupled with the Background

$F_{1-loop}(\Phi=1)$

$$\sim - \frac{4N_F (4\pi b^2) V \cdot 2 \left[ \frac{D-2}{2} \right]}{(4\pi)^{D/2}} \Gamma\left(\frac{D}{2}\right)$$

$$\cdot \left[ \left(1 - \frac{1}{2^{D-1}}\right) \zeta(D) \left(\frac{2}{\beta}\right)^D - \frac{1/b^2}{3(D-2)} \left(1 - \frac{1}{2^{D-2}}\right) \zeta(D-2) \left(\frac{2}{\beta}\right)^{D-2} \right]$$

$F_{1-loop}(\Phi=0)$

$$\sim - \frac{4N_F (4\pi b^2) V \cdot 2 \left[ \frac{D-2}{2} \right]}{(4\pi)^{D/2}} \Gamma\left(\frac{D}{2}\right)$$

$$\cdot \left[ \left(1 - \frac{1}{2^{D-1}}\right) \zeta(D) \left(\frac{2}{\beta}\right)^D + \frac{2/b^2}{3(D-2)} \left(1 - \frac{1}{2^{D-2}}\right) \zeta(D-2) \left(\frac{2}{\beta}\right)^{D-2} \right]$$

$$F_{chiral}(\Phi=0) = \frac{1}{4\pi^2 b^4} \times V \cdot (4\pi b^2)$$

if  $F_{1-loop}(|\Phi|=1) - F_{1-loop}(\Phi=0) > F_{chiral}(\Phi=0)$

Phase tr. could occur.

$$T = \beta^{-1} > \frac{1}{2} \left[ \frac{(4\pi)^{D/2} (D-2) \left(1 - \frac{1}{2^{D-2}}\right)}{16 N_F b^2 \cdot 2 \left(\frac{2}{\beta}\right)^2 \Gamma(D/2)} \right]^{\frac{1}{D-2}}$$

$N_F$ : # of Fermion doublets

§5. INFLATION ?

$F_{1-loop}$

$$= \frac{N_F \cdot V \cdot 2^{(L+1)} \cdot \beta}{(4\pi)^{L+1}} \sum_{n=-\infty}^{\infty} \sum_{l=0}^{\infty} d(l) \left[ M^2(l) + \left(\frac{2\pi n}{\beta}\right)^2 \right]^{L+1}$$

$d(l) = 4(l+1)$ ,  $M^2(l) = \frac{(l+1)^2}{b^2}$  for  $|\Phi|=1$  (TRIVIAL VAC)

$d(l) = 2(2l+1)$ ,  $M^2(l) = \frac{2(l+1)}{b^2}$  for  $|\Phi|=0$  (MONOPOLE B.G.)

(Here we don't consider

"KINEMATICAL" inflation

cf. K-K inflation)

effective potential for scale factor  $b$



$D \geq D_c$

$D < D_c$

$D_c = 15$  for  $\beta = 1$  (Miyamoto, Yamamoto, 1982)  
 $D_c = 28$  for  $\beta = 10$  (Miyamoto)

$D=8$  (flat 4-dim Misner-like s.t.)

$$\delta \mathcal{L} = -\frac{q}{4} (R_{min}^2 - 4R^2 + R^3)$$

$0 < q < \frac{1}{40}$  for "Domain wall" model

But exponent of expansion factor

$\sim H^2 b^2 \sim 0.4$  - almost 0 for  $\beta = 10$

50-60

slow roll-over?



$$\ddot{\phi} + 3H\dot{\phi} - m^2\phi = 0$$

$\ddot{\phi} \ll 3H\dot{\phi}$  (slow change)

$$\phi \propto \exp\left(\frac{m^2}{3H}t\right)$$

TIME SCALE FOR ROLLING-DOWN

$$\tau \sim \frac{3H}{m^2}$$

EXPANSION Factor

$$= \exp H\tau \approx \exp \frac{3H^2}{m^2}$$

IN THE PRESENT MODEL

$$m^2 \sim \frac{1}{p^2}$$

$$\delta \mathcal{L} = \frac{1}{4g^2} G_{\mu\nu} G^{\mu\nu} \text{ is "vector" model}$$

UV gauge field on  $S^2$

$$G_{mn} \propto \epsilon_{mn} \quad q^i = \frac{v^i q^2}{2g^2} \quad (g \rightarrow 0, \text{ "instanton"})$$

if  $q^2 \gg 1$

$$2\Lambda n^i \sim \frac{v^i q^2}{2g^2 B^i} \sim \frac{1}{B^i}$$

$$\therefore B^i \sim \frac{v^i n^i}{2g^2}$$

$$\frac{n^i \Lambda q^i}{g^2} \sim 1$$

effective pot. for  $b$   $\hookrightarrow$  OK

But expansion factor  $\sim \exp \frac{1}{p^2} !!$



# § 6. Conclusion

• 二二二の簡単なEYM-like (E-Y-M action)

Instanton - Induced Compactification  $U(2) \times U(1) \rightarrow$

"trigon" to First Compactification

$$dX \sim \frac{1}{\sqrt{2}} G_{\mu\nu}^a \text{ etc.}$$

• rolling-down solution is  $U(2) \times U(1)$ .

$$c \quad \rho^2 \approx 2 \times 10^{-16} \text{ cm}$$

↓ solve it.

• in flat 3 dim space ... we live "outside" of topological defects

"vortex" "Domain walls"

• Quantum Effect.

viscosity from particle creation



variation of "field strength"

$$D_{\mu\nu} F^{\mu\nu} = 0 \rightarrow dS^2 = R_{\mu\nu}^2 dx^{\mu} dx^{\nu} + dr^2 + R^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$\cdot -\frac{1}{R^4} \partial_r (R^4 \partial_r A_t) + 2 \frac{1}{R^4} (1+A_t)^2 \dot{\Phi}^2 = 0$$

$$\cdot \partial_r^2 \Phi + \frac{2}{R} \partial_r \Phi + d \frac{\partial R}{R} \partial_r \Phi - \frac{1}{R^4} (1+A_t)^2 \dot{\Phi} + \frac{1}{R^2} (1-\Phi^2) \dot{\Phi} = 0$$

$$R_{\mu\nu} = \kappa^2 (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) + \frac{\kappa^2 \Lambda}{3} g_{\mu\nu}$$

$$\cdot \frac{\kappa^2}{D-2} \left[ 2\Lambda - \frac{1}{2\sigma^2} \left\{ \frac{2}{b^2} (2a\dot{\Phi})^2 + \frac{1}{a^2} (1+A_t)^2 \dot{\Phi}^2 \right\} + \frac{1}{b^2} (2A_t)^2 + \frac{1}{b^2} [(1-\Phi^2)^2 + \dot{\Phi}^2] \right] g_{\mu\nu}$$

$$= R_{\mu\nu} = - \left[ \left( \frac{R}{a} \right)' + \left( \frac{R}{b} \right)' \right] + (1-1) \left( \frac{R}{a} - \left( \frac{R}{b} \right)' \right) - \frac{A_t R'}{R}$$

$$- 2 \left[ \left( \frac{R}{a} \right)' + \left( \frac{R}{b} \right)' \right] - d \left[ \left( \frac{R}{a} \right)' + \left( \frac{R}{b} \right)' \right] + 2 \frac{A_t}{R} \dot{\Phi} + d \frac{R}{R} \dot{\Phi}$$

$$= \frac{\kappa^2}{\sigma^2} \left\{ \frac{1}{b^2} (2\dot{\Phi})^2 - \frac{1}{a^2} (1+A_t)^2 \dot{\Phi}^2 \right\}$$

$$\cdot - 2 \left[ \left( \frac{R}{a} \right)' + \left( \frac{R}{b} \right)' \right] - d \left[ \left( \frac{R}{a} \right)' + \left( \frac{R}{b} \right)' \right] + 2 \frac{R}{R} \dot{\Phi} + d \frac{A_t}{R} \dot{\Phi}$$

$$- 2 \frac{1}{b^2} + 2 \left( \frac{R}{b} \right)' = \frac{\kappa^2}{\sigma^2} \left[ \frac{1}{a^2} (2A_t)^2 - \frac{1}{b^2} [(1-\Phi^2)^2 + \dot{\Phi}^2] \right]$$

$$\cdot \frac{1}{b^2} - \left( \frac{R}{a} \right)' - \left[ \left( \frac{R}{b} \right)' + \left( \frac{R}{a} \right)' \right] - \frac{A_t}{R} \dot{\Phi}$$

$$= \frac{\kappa^2}{D-2} \left[ 2\Lambda + \frac{1}{\sigma^2} \left\{ \frac{2}{b^2} (2a\dot{\Phi})^2 + \frac{2}{a^2} (1+A_t)^2 \dot{\Phi}^2 + \frac{2}{b^2} [(1-\Phi^2)^2 + \dot{\Phi}^2] - \frac{1}{2\sigma^2} (2A_t)^2 \right\} \right]$$